Lesson 1 What is Mathematics? Mathematics History

Three explorers descend from the clouds in Katmai National Park, Alaska. God gave us math as a tool to explore, discover, and know truth. For example, we see 3 explorers in the photo, not 4, or 7, or any other number. Only the numeral 3 truthfully describes how many explorers are in the photo. God is the ultimate source of truth. He reveals truth to us primarily through the Bible, but also through His creation.

Rules

Skip Counting, 1's to 12's, plus 25's (memorize as much as you can, and more!)

1's: 1, 2, 3 ,4, 5, 6, 7, 8, 9, 10, 11, 12...
2's: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24...
3's: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36...
4's: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48...
5's: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60...
6's: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72...
7's: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84...
8's: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96...
9's: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108...
10's: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120...
11's: 11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132...
12's: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144...
25's: 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275...

Definitions

sequence: Numbers ordered in a way that they form a definite pattern.

1A What is Mathematics?

Welcome to *Shormann 6*, I'm glad you're here! Let's start by answering the question, "What is mathematics?" Well, mathematics, or "math" is actually many things! In Shormann Mathematics, we break math into 10 topics, listed below. Some topics have long names, but that's okay! All you need to do right now is read the words in the list below.

10 Topics in Shormann Math
Number
Ratio
Algebra
Geometry
Analytical Geometry
Measurment
Trigonometry
Calculus
Statistics
Computer Mathematics

Did you know that male and female humans, and no other life forms, were created in God's image (Genesis 1:26)? He designed us to be creative, like Him! As the famous scientist, Johannes Kepler said, when we study His creation, we "think God's thoughts after Him."

God gave us the ability to manage His creation (Genesis 1:28). He wants us to use the things He made in responsible ways that show we are thankful for what He has provided. God gave us mathematics as one of many tools to manage His creation. Here is how we define mathematics in all our *Shormann Math* courses:

Mathematics is the language of science and a God-given tool for measuring and classifying pattern and shape.

Dr. Shormann

Most math courses avoid talking about God, which is strange! The Bible says we are to acknowledge Him in all our ways (Proverbs 3:6), which means all the things we do, including doing math, are connected to God.

The Bible also says that God is truth (John 14:6). Math reveals truth, which is why we define math as a God-given tool. Because math reveals truth to us, it points us to God, the source of truth.

I hope this course helps you learn math! But, there are more important things than math. Like salvation. The Bible says that all of us are sinners and fall short of His glory (Romans 3:23). Without God, we are wicked and condemned to eternal death. But God sent His son, Jesus Christ, who is one with God (John 10:30), to pay the debt of our sins, promising eternal life for all who believe (John 3:16).

If you are not a Christian, that's okay! But, if you want to become a Christian, simply believe Jesus can save you and repent of your sins, then be baptized "in the name of Jesus Christ, for the forgiveness of your sins, and you will receive the gift of the Holy Spirit." (Acts 2:38-39). There is only one way to Heaven, and it's through God's son, Jesus Christ. No sin is too great, nothing you have done is too bad to receive Christ's forgiveness. I am glad Jesus saved me!

1B Sequences

In math, we use sequences as one way to classify patterns. Study the skip-counting chart in the Lesson 1 Rules. Memorizing sequences like these will make math easier, guaranteed!

Think about any game you have ever learned to play. What is the first thing you have to do? Know the rules! And the game gets easier once you have the rules memorized, right? Math is no different, so do your best to memorize new Rules and Definitions. Over time, hard things in math get easier as you memorize Rules and practice playing the game of math.

For sequences, we will do problems like the following example:

Example 1.1 Fill in the missing numbers in the following sequence:

3, 6, 9, ___, ___, ____, 21, 24, 27, 30, 33, 36...

solution: If you have this sequence memorized, great! If not, study the skip-counting chart in the Rules and see that the missing numbers are **12, 15, and 18**.

1C Mathematics History

Math history begins after the Genesis Flood, with the Egyptians and Babylonians around 2500 B.C. (2,500 years before Christ's birth). "B.C." means "Before Christ." It is hard to say what knowledge was passed to the Babylonians from the pre-Flood world, but since Noah and his family were shipbuilders, they probably had extensive knowledge of many math topics. A lot of math is involved in building a ship, especially one that was 300 cubits (450 feet) long (Genesis 6:15).

In part 1A, we defined math as a "God-given tool." God is truth, and math leads us to truth, so it makes sense that God made math. He designed human beings to discover, and use, what he made. Imagine if we were born with the ability to do math. That would actually be really boring!

Instead, God gave us the ability to explore, discover and know. He also gave us the ability to choose. You can choose to be really boring and lazy, or you can choose to explore and discover and ask questions and try to know Him and His amazing purpose for you.

We could spend A LOT of time on math history, but we will wait and do that more in future courses. For now, look at this quote from Leonhard Euler (1707-1783), considered the best mathematician in history:

It is God therefore, who places men, every instant, in circumstances the most favorable, and from which, they may derive motives the most powerful, to produce their conversion; so that men are always indebted to God, for the means which promote their salvation.

Letters to a German Princess, V. 1, p. 510

Sadly, almost no one today knows that the best mathematician in history was a Christian. But, now you know! The quote is from a letter Euler wrote to a German princess with questions about things like math, science and Jesus. What Euler is saying is that God uses any means necessary, including math, to point people to Jesus and save them!

This amazing man, considered by scholars as the best mathematician ever, was concerned about the eternal destiny of his students. Following Christ is more important than math. The best mathematician in history knew this. Do you?

Practice Set 1

1. God designed us to be ______________like Him!

2. What Bible verse teaches us that God gave us the ability to manage His creation?

3. Which of the following is how we define mathematics in Shormann Math?

    A) A study of space and quantity.
    B) A God-given tool for measuring and classifying pattern and shape.
    C) The science which investigates the means of measuring quantity.
    D) The classification and study of all possible patterns.

4. Because math reveals ___________to us, it points us to God.

5. The Bible says that all of us are ____________ and fall short of His glory.

6. Fill in the missing numbers: 10, 15, ___, ___, ____, 35, 40...

7. Fill in the missing numbers: 15, ___, ___, ____, 27, 30, 33...

8. Fill in the missing numbers: 13, 14, 15, ___, ___, ____, 19...

9. Fill in the missing numbers: 12, 24, 36, ___, ___, ___, 84...

10. Fill in the missing numbers: 22, 24, 26, ___, ___, ____, 34...

11. Since __________and his family were shipbuilders, they probably had extensive knowledge of many math topics.

12. God gave us the ability to ___________, discover and know.

13. You can ____________ to be really boring and lazy, or you can go explore and discover and ask questions and try to know Him and His amazing purpose for you.

14. Sadly, almost no one today knows that__________, the best mathematician in history, was a Christian.

15. Dr. Shormann believes following Christ is more important than _________, and hopes you believe that, too!

Lesson 2 Thinking About Number; Place Value to the Trillions

Legendary math teacher John Saxon (1923-1996) said "a number is an idea." A numeral is something we write that expresses the idea of a particular number. For example, How many killer whales do you see in the photo above? What lesson number is this? For both questions, the answer is "2" or "two". There are lots of other things that represent "twoness," like two eyes, two ears, 2 is after 1 and before 3, etc. We can use the idea of one number to describe many things!

Rules

Comparison symbols

symbol	name
=	equals
>	greater than
<	less than

Place Value Chart

hundred trillions	ten trillions	trillions		hundred billions	ten billions	billions		hundred millions	ten millions	millions		hundred thousands	ten thousands	thousands		hundreds	tens	ones		tenths	hundredths	thousandths
___	___	___	,	___	___	___	,	___	___	___	,	___	___	___	,	___	___	___	.	___	___	___

Definitions

number: An idea representing a quantity.
numeral: Any symbol used to describe a number.
digit: Any of the Hindu-Arabic numerals 1 through 9, and 0.
place value: A digit's value, based on its placement within the numeral.

counting (natural) numbers: Numbers used to count objects. Ex. 1, 2, 3, ...
whole numbers: Counting numbers and the number 0.
integers: Whole numbers, and all negative numbers that are not fractions or decimals.
real numbers: Any number used to describe a positive or negative number. Includes all integers, and all decimal numbers and fractions.

$\frac{2}{3}$ 1.2234 -17 43.11 $3\frac{1}{8}$

zero: A number that is neither positive or negative.
negative numbers: Numbers less than zero.
postive numbers: Numbers greater than zero.

2A Thinking About Number

In Lesson 1, we defined math as a God-given tool that helps us classify patterns. Sequences were the first math pattern we described. In Lesson 2A, we will cover some definitions we use to classify numbers. As you learn the different ways numbers are classified, notice the importance of zero. For example, the difference between whole numbers and counting numbers is that whole numbers include zero. Or note how positive numbers are above (greater than) zero, while negative numbers are below (less than) zero. We also use number lines to help us compare numbers, and we often draw number lines with zero in the middle. If you have zero of something, you have nothing. But even though zero is nothing, it's still really important!

Example 2.1. Which set of numbers begins with 0?

A) natural numbers B) whole numbers C) integers D) real numbers

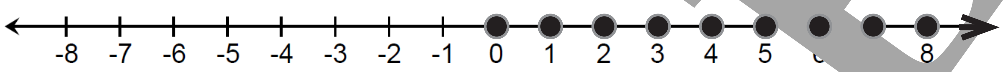
solution: Whole numbers, Choice B. Notice also, in the question, we described the number types as *sets* of numbers. We classify number types into *sets* or *groups*.

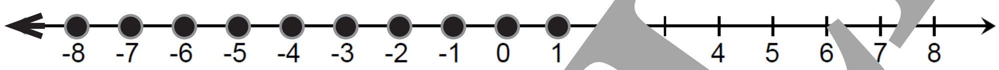
Example 2.2. Which number type includes fractions and decimals?

A) counting numbers B) whole numbers C) integers D) real numbers

solution: Real numbers, Choice D. Real numbers are the only set of numbers that includes fractions and decimals. This is an important distinction because, any time you see a decimal number or a fraction, you know it cannot be a natural (counting) number, a whole number, or an integer. Memorize the different number types!

Example 2.3. Which number line represents counting numbers?

A) 

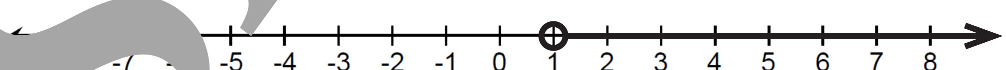
B) 

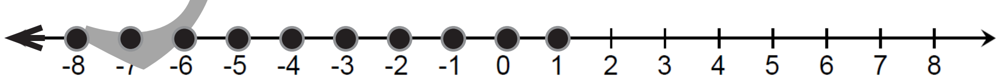
C) 

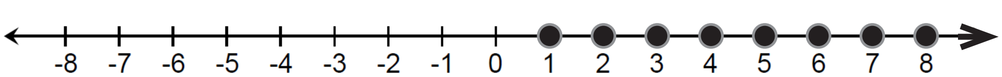
D) 

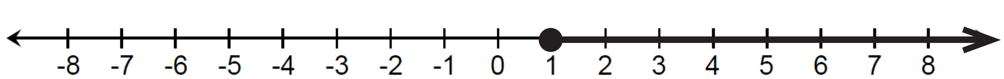
solution: Counting or natural numbers start with 1, and increase from there, one at a time, 2, 3, 4, ... On a number line, dots (closed circles) are used to represent specific numbers. Choice A starts with a dot at zero, and Choice B also has dots identifying negative numbers. Both are therefore incorrect. **Choice C** is correct because it starts at 1 and has dots at 2, 3, etc. Choice D starts at 1, but it has a solid line, which means it rep resents ALL numbers from 1 and up. Choice D therefore represents real numbers greater than or equal to 1, not counting numbers.

Example 2.4. n≥1 means "real numbers greater than or equal to 1." Which number line represents n≥1?

A) 

B) 

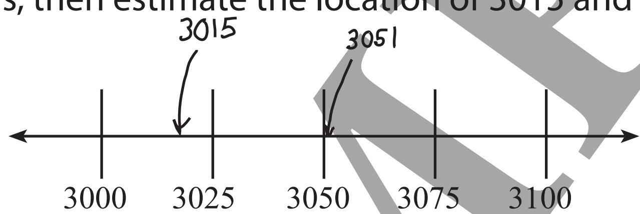
C) 

D) 

solution: On a number line, we use open circles (○) to represent "right next to, but not equal to" a number. Closed circles (●) represent "equal to" a number. Therefore, Choice A represents n>1, but not equal to 1. Choice B represents integers and Choice C represents counting numbers (can also represent positive integers). **Choice D** is correct, as it represents n≥1.

Example 2.5. Compare 3015 □ 3051. Use a number line to help you.

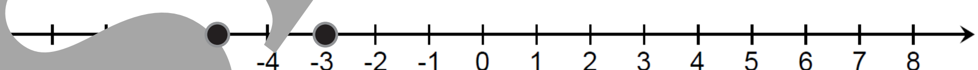
solution: When you are asked to compare two numbers, you are being asked to compare if one nunber is greater (>), less (<), or equal to (=) the other number. You may be able to tell that **3015 < 3051**, but if not, mapping the numbers on a number line may help. There is no specific way to make a number line, but obviously we don't want to start at 0 and mark numbers out to 3051! Instead, lets start at 3,000 and skip count (Lesson 1) by 25's, then estimate the location of 3015 and 3051:



On a number line, numbers to the left are less than (<) numbers to the right, confirming that **3015 < 3051**.

Example 2.6. Compare -3 □ -5. Use a number line to help you.

solution: As mentioned in Ex. 2.5, **on a number line, numbers to the left are less than numbers to the right**. If we plot -3 and -5 on a number line, -5 appears to the left of -3, meaning -5 is less than(<) -3:



Alternatively -3 is greater than -5, so to compare -3 □ -5, we write **-3>-5**.

2B Place Value to the Billions

The value of each digit in our number system depends on its position in a number. We call this the digit's *place value*. Each digit is called a numeral. We use the Hindu-Arabic numeral system, which includes numerals 1-9, plus 0.

Example 2.7. For each number shown, what is the numerical value of the 3?

a) 2,300 b) 5.35 c) 36,000,000,000,000

solution: The value of a digit in a number is found with two steps:

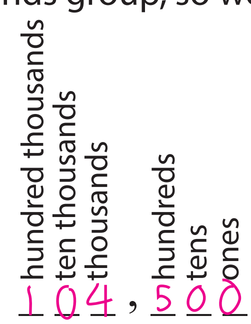
1) Use the Place Value Chart to identify the digit's place value.
2) Add *placeholder zeros* to the left or right, until you reach the decimal point. Placeholder zeros are just that, they "hold" the place of the digit in question so that you can correctly write its numerical value.

a) The 3 is in the hundreds place, so the value is **300**.
b) The 3 is in the tenths place, so the value is **0.3**.
c) The 3 is in the ten trillions place, so the value is **30,000,000,000,000**.

Example 2.8. Use words to write the number 104500.

solution: When using words to write numbers, we put commas every three places, so after *trillions, billions, millions, and thousands*. Including commas every 3 digits in numbers is optional but recommended, so we will rewrite 104500 as 104,500. We can also write the digits into our place value chart, to help us see the 104 is in the thousands group, so we write 104 as one hundred four thousand.

We don't use "and" when writing numbers, unless there's a decimal place. "One hundred *and* four thousand" is incorrect. The last three digits are 500, or five hundred. Therefore,



104,500 = one hundred four thousand, five hundred.

Example 2.9. Use digits to write the number five million, three hundred ten thousand, four hundred sixteen.

solution: Again, if it helps, use the place value chart to help you properly position digits. Also notice the comma placement in the words:

Therefore, using digits we write **5,310,416**.

Practice Set 2

The subscripted number next to the problem number references which lesson the problem is from.

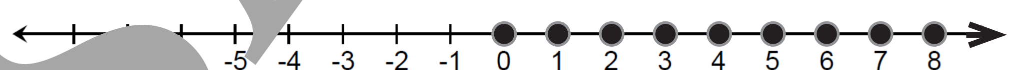
1_2. Which set of numbers begins with 1?

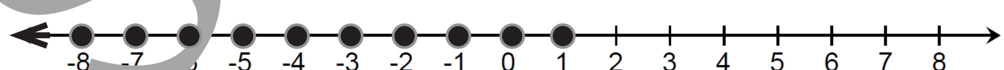
A) counting numbers B) whole numbers C) integers D) real numbers

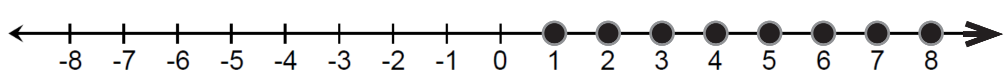
2_2. Which of the following numbers is an integer?

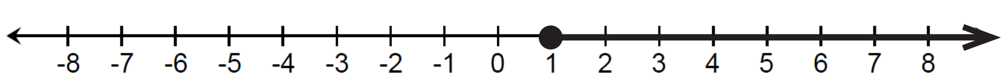
A) 3.2 B) -3.2 C) -7 D) 1/3

3_2. Which number line represents whole numbers?

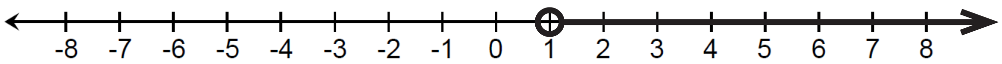
A) 

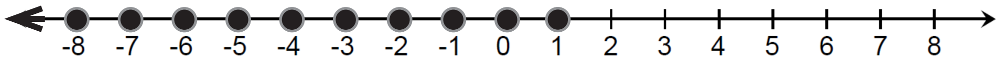
B) 

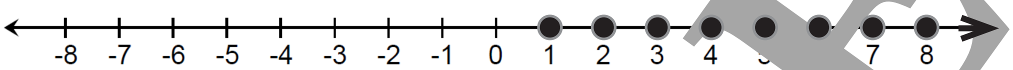
C) 

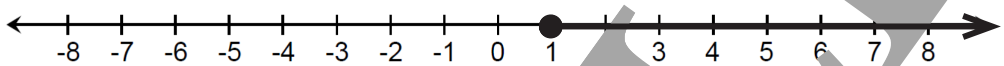
D) 

4_2. n>1 means "real numbers greater than1." Which number line represents n>1?

A) 

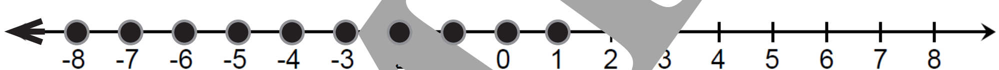
B) 

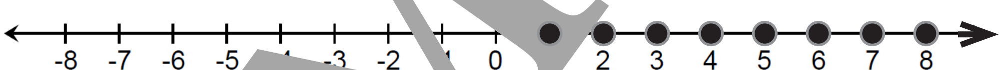
C) 

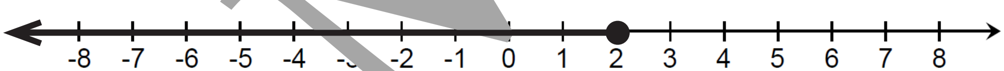
D) 

5_2. In math, the symbol $\mathbb{Z}$ might be used to represent integers. Which number line represents $\mathbb{Z}$ <2, the integers less than 2?

A) 

B) 

C) 

D) 

6_2. Compare 5100 □ 5099. Optional: Use a number line to help you.

7_2. Compare 23,175 □ 23,057. Optional: Use a number line to help you.

8_2. Compare -3 □ -1. Optional: Use a number line to help you.

9_2. Compare 1 □ -7. Optional: Use a number line to help you.

10_2. What is the numerical value of the 5 in 251,318?

11_2. What is the numerical value of the 7 in 1.117?

12_2. Use words to write the number 1080005.

13_2. Use digits to write the number twenty thousand, five hundred nineteen.

14_2. Use digits to write the number eleven million, four hundred seventy thousand, six.

15_1. In Shormann Math, we define math as the language of ________, and a God-given tool for measuring pattern and shape.

16_1. Fill in the missing numbers: 50, 75, 100, _____, _____, _____...

17_1. Fill in the missing numbers: 6, _____, _____, _____14, 16...

18_1. Fill in the missing numbers: 21, 28, _____, _____, _____, 56...

19_1. God could have made us like boring robots, but instead he gave us the ability to ____________, discover and know.

20_1. Euler, the best mathematician ever, also said that "men are always indebted to God, for the means which promote their _________________."

Lesson 3 Addition of Whole Numbers and Decimals

Rules

\$ = dollar sign Used to indicate a value in whole dollars, or dollars and cents. Written at the front (to the left) of the number. For example, \$25, \$10,366.28.
¢ = cent symbol Used to indicate a value in cents. Written at the end (to the right) of the number. For example, 50¢, 236¢.
+ = plus sign Used to indicate addition of two or more numbers.

Associative property for addition: $(a + b) + c = a + (b + c)$
When you have more than two numbers to add, the way you group them in pairs does not matter. Example: $(2+3)+4 = (5) + 4 = 9$, and $2+(3+4) = 2+(7) = 9$

Commutative property for addition: If $a + b = c$, then $b + a = c$
Order does not matter. Example: $2+3 = 5$, and $3 + 2 = 5$

Additive identity: $a+0 = a$. Any number plus zero equals that number.

Definitions

addend: A number being added.
sum: The result of addition. For example, *addend + addend = sum*
arithmetic: A name that is normally used to describe the four operations of addition, subtraction, multiplication and division.
fact family: Three numbers arranged in different orders to create four arithmetic facts. Fact families help us understand the relationship between addition and subtraction, as well as multiplication and division.

In Lesson 1, we learned that math is about finding patterns. In Lesson 2 we looked at the pattern of place value. In Lesson 3, we will review addition of whole numbers and decimal numbers that have the *same* number of decimal places. Addition with *different* numbers of decimal places will be covered later.

Did you notice how Lesson 3 builds on Lesson 2, which builds on Lesson 1? This is an amazing thing about math, that learning new things depends on past things. This is why it's important to memorize Rules and Definitions, as best as you can!

In addition, the main pattern to notice is that you line up place values, then sum all the digits in a single place value, carry digits in the sum that are in the tens place, then add the next place value. Simply repeat the same pattern until you are done.

Example 3.1. Add the following pairs of whole numbers.

a)	b)	c)
30	24	6,819
+0	+17	+4,504

solution: Note the numbers are already lined up by place value, so just add the digits in each place, and if the digits sum to 10 or more, write the digit in the ones place beneath the addition line, and "carry" the 1 to the column to the left. By "carry" we mean "add the 1 to the sum of the digits in the next column."

a)	b)	c)
30	24	6,819
+0	+17	+4,504
30	**41**	**11,323**

Note that in a) we applied the additive identity from the rules. It was also probably the easiest addition problem you could do! We also had no digits to carry. In b) we carried 1 from the ones place because 4+7=11. We had to carry twice in c). Note alsoi in c) how, when we added 6+4+1=11, but there was no ten thousands place in the addends to carry the 1 to. Therefore, we finished by writing 11 below the addition line.

For all three problems, we call these groups of three numbers *fact families*. We will talk more about fact families in later lessons, but note how each problem has two addends and a sum. Note also how, according to the *commutative property for addition*, we can arrange the addends in two ways and still get the same answer. For example, we could have written 3.1b as 24+17=41, or 17+24=41. Each pair of addends can be written as two addition facts.

Example 3.2. Add the following whole numbers.

a) 1+1+2+3+5
b) 1,481+82+783

solution: For a) the numbers are just single digits, so the simplest thing to do is add in pairs, record those results, and repeat. Per the *associative* *property for addition*, it does not matter what pairs we add. Normally we will just start on the left on a problem like this. We will use parentheses to group in pairs:

```
    (1+1)+(2+3)+5=
      (2 + 5) +5=
         (7 + 5)=
a)          12
```

For b) line up the place values first, as seen in Ex. 3.1, then add. Note how the largest number is on top and the smallest is on bottom:

```
    1 2
    1,481
      783
   +   82
b)  2,346
```

Example 3.3. Add the following pairs of real numbers.

```
                          10,487.6
a) 1.035+4.907      b) +     333.3
```

solution: Add real numbers just like whole numbers. For a), line up the place values and add. One way to know if you lined things up correctly is that the decimal points should be aligned:

```
        1            1 1
      1.035       10,487.6
   +  4.907    +     333.3
a)    5.942    b) 10,820.9
```

Example 3.4. Add the following real numbers.   6.5+10.9+104.8

solution: As in Ex. 3.3a, line up the decimal points as an easy way to make sure you are lining things up correctly. Largest number on top as in Ex. 3.2b:

```
     1 2
     104.8
      10.9
   +   6.5
     122.2
```

Example 3.5. Add.

$$\begin{array}{r} \$79.95 \\ + \ \$14.50 \\ \hline \end{array}$$

solution: The only difference between Ex. 3.5 and previous examples is the "$" symbol in front of the numbers. The symbol is mainly used to indicate money amounts in United States dollars. The way we add money and other real numbers is identical:

$$\begin{array}{r} \$79.95 \\ + \ \$14.50 \\ \hline \mathbf{\$94.45} \end{array}$$

Example 3.6. Joe's family ate out for $127.49. He added a tip of $19.13. How much did Joe pay altogether?

solution: We call problems like this *word problems*, or *story problems*. In this problem, the story is that Joe's family ate out, and we want to know how much he paid altogether. Word problems have clues that help us know how to solve them. Our first clue is the word "added," which directs us to add the meal plus the tip. The word "altogether" also directs us to use the two money values given and combine them. A third clue is this problem is from Lesson 3 on addition, so let's line up decimal points and add:

$$\begin{array}{r} \$127.49 \\ + \quad \$19.13 \\ \hline \mathbf{\$146.62} \end{array}$$

Practice Set 3

The subscripted number next to the problem number references which lesson the problem is from.

1_3. Add the following pair of whole numbers. $\begin{array}{r} 83 \\ +54 \\ \hline \end{array}$

2_3. Add the following pair of whole numbers. $\begin{array}{r} 2{,}441 \\ +5{,}680 \\ \hline \end{array}$

3_3. Add the following whole numbers. 2+0+5+1+3

4_3. Add the following whole numbers.   318+27+4,055

5_3. Add the following pair of real numbers.   2.095+9.913

6_3. Add the following pair of real numbers.

$$\begin{array}{r} 49{,}965.4 \\ +\quad 327.8 \\ \hline \end{array}$$

7_3. Add the following real numbers.   2.1+12.1+112.1

8_3. Add.

$$\begin{array}{r} \$99.95 \\ +\ \$21.99 \\ \hline \end{array}$$

9_3. Andrea used her debit card to get $200.00 out of her account. The ATM machine charged a transaction fee of $3.75. What was the total cost of Andrea's transaction?

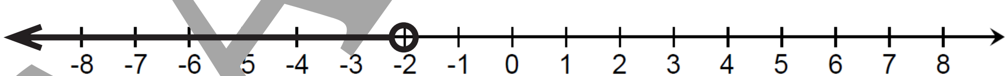
10_2. Which set of numbers includes -1?

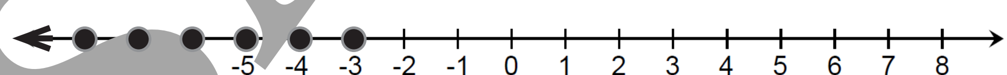
A) natural numbers   B) whole numbers
C) integers   D) counting numbers

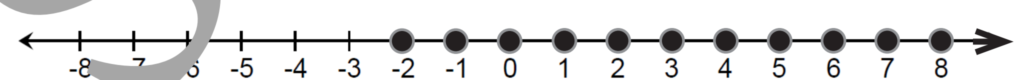
11_2. Which set of numbers is the same thing as counting numbers?

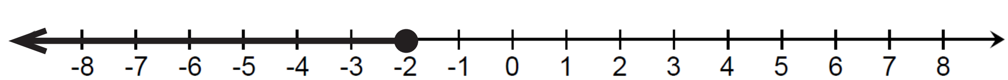
A) negative numbers   B) whole numbers
C) integers   D) natural numbers

12_2. n<2 means "real numbers less than 2." Which number line represents n<2?

A)


B)


C)


D)


13_2. Compare 107 □ 135. Optional: Use a number line to help you.

14_2. Compare -1 □ -6. Optional: Use a number line to help you.

15_2. What is the numerical value of the 7 in 6.07?

16_2. Use words to write the number 550,000,000,000,000.

17_2. Use digits to write the number six thousand, three hundred six.
18_1. God designed human beings to be __________, like Him.

19_1. The following pattern shows skip counting by ______.

20_1. Math history begins after the Genesis Flood, with the Egyptians and Babylonians around _________ B.C.

Lesson 4 Subtraction of Whole Numbers and Decimals; Order of Operations(Parentheses)

Rules

minus sign The "-" symbol. Used to indicate subtraction of two numbers.
PEMDAS is an acronym often used to describe the correct order of operations, where inside **P**arentheses and **E**xponents are simplified first, followed by **M**ultiplication/**D**ivision, and finally **A**ddition/**S**ubtraction.

Definitions

minuend: The first number in subtraction. Usually the larger number.
subtrahend: The second number in subtraction.
difference: The result of subtraction. For example, *minuend - subtrahend = difference*

Lesson 4 will have problems like you found in Lesson 3, with a focus on subtraction. We will also introduce order of operations and the PEMDAS acronym that helps us remember what operations to do first. In Lesson 3, you learned that order does NOT matter in addition, but in many other situations, order makes a big difference!

For problems involving both addition and subtraction, an important rule to remember is to **simplify inside parentheses first**. First though, let's explore fact families more, and how they can help us understand the relationship between addition and subtraction.

You might be thinking "addition and subtraction are different things, how can they be related?" Well, go back to Lesson 1. Who made math? God did, and He is relational. The Bible, God's word, says God is love (1 John 4:8). And love requires a relationship, which also tells us the Father, Son and Holy Spirit have always existed, forever! The Bible doesn't say God became love, or God wasn't love until the Son was born, etc. It says God IS love.

So, if God has always been relational, and God made math, it makes sense that math would express His character, which is why we find relationships between different things like addition and subtraction! Just like there is never a time when God is not love, there is never a time when addition is not related to subtraction. One of the most beautiful things about math is that it reveals God's nature to us. Even little things, like the relationship between addition and subtraction, can point us to Him!

Example 4.1. 3+2=5 is a fact family. List three more fact families using these numbers, 1 addition and two subtraction facts.

solution: In Lesson 3, we defined fact families as 3 numbers we can arrange to make 4 arithmetic facts. When addition and subtraction are involved, we can make two addition facts and two subtraction facts. For example, the commutative property for addition (Lesson 3) tells us order doesnt matter for the addends, so one more addition fact is **2+3=5**. Moving to subtraction, we can see the minuend (largest number) in the fact family is 5, so we can write **5-3=2**, and **5-2=3** as the two subtraction facts. Fact families work in groups of 4 like this, which is really interesting! It is also a good reminder that fact families can help us check work on an addition or subtraction problem, which you will see next. Finally, the word “family” is used here because there is an obvious relationship between addition and subtraction. This also points us back to God, who wants a personal relationship with each person.

Example 4.2. 4-1=3 is a fact family. List three more fact families using these numbers, 1 subtraction and two addition facts.

solution: Order does not matter in addition, but it DOES matter in subtraction! The minuend (largest number) must go first. So, one other subtraction fact is **4-3=1**. The two addition facts that complete this family are **1+3=4** and **3+1=4**.

Example 4.3. Subtract the following pairs of whole numbers.

a) $\begin{array}{r} 24 \\ \underline{-17} \end{array}$ b) $\begin{array}{r} 6{,}819 \\ \underline{-\ 4{,}504} \end{array}$

solution: In addition, we sometimes need to carry to the next place value. With subtraction, we sometimes need to borrow, when one of the minuend’s digits is smaller than the subtrahend’s below it.

a) $\begin{array}{r} {}^{1}\not{2}{}^{1}4 \\ \underline{-\ 17} \\ \mathbf{7} \end{array}$ The 2 is in the tens place, so we “borrow 10” and add it to the 4 to get 14. IMPORTANT: We just said the little 1 above the 4 makes this a 14. That is different than addition, when the handwritten digit is just that, a single digit, and does not mean “add 10.” **Please notice this difference between addition and subtraction, and how the little handwritten numbers we write have different meanings, depending on whether we are adding or subtracting.** Since we borrowed 10, we have to take 10 away from 20, which is 20-10=10, so we cross out the 2 and replace it with a 1. We then subtract 14-7=7, then move to the tens place and subtract 1-1=0, which we don’t write. Note how 24, 17 and 7 \ make a fact family. We can use the fact family idea to check our work, by adding 17+7=24. Also, note that 7+17=24, and 24-17=7.

```
    6,819
  - 4,504
b)  2,315
```

We didn’t have to borrow like in a). Using fact families, we can check our work by adding 2,315+4,504:

```
  2,315
+ 4,504
  6,819
```

Example 4.4. Add and subtract. Simplify inside parentheses first.

a) (2-1)+4+6-3
b) 6+2+1+(5-3)-2

solution: First, we simplify inside parentheses. Second, we **move left to right,** adding and subtracting in pairs. We will group pairs inside parentheses.

a) (2-1)=1, so rewrite the problem with the 1, move left to right, and add/subtract in pairs:
1+4+6-3=(1+4)+(6-3)=
5 + 3 = **8**

b) (5-3)=2, so rewrite the problem with the 2, move left to right, and add/subtract in pairs:
6+2+1+2-2= (6+2)+(1+2)-2=
(8 + 3) - 2=
11 - 2= **9**

Example 4.5. Subtract the following pairs of real numbers.

a) $3.67-$1.99 b) 4,000.2 − 1,983.3

solution: Subtract real numbers just like whole numbers. For a), line up the place values and subtract. One way to know if you lined things up correctly is that the decimal points should be aligned. Check your work using an addition fact:

a) $3.67-$1.99

```
   $3.67              $1.99
-  $1.99           +  $1.68
   $1.68              $3.67
```

Check: Note with the 6 and 7, we had to borrow 1 from the 6 and make it a 5, then subtract 17-9=8. Then, in the tenths place, we had 5-9, so we had to borrow 1 from the 3 and make it a 2, then subtract 15-9=6.

```
    4,000.2           2,016.9
-   1,983.3         + 1,983.3
b)  2,016.9           4,000.2
```

Check: We can't subtract 2-3, so we have to borrow 10 to make 12. But, the next three places are 0's, so we have to borrow 1 from the 4 and make it 3, then add to the 0 in the hundreds place to get 10, borrow 1 from the 10 and make it a 9, and continue the pattern until we get 12. Note that you CANNOT borrow 1 from the 4 and give it to the 2, skipping over place values. Think about it, borrowing 1 from the 4 is like borrowing 1,000, since it is in the thousands place. So, you are really borrowing 1,000, which you borrow 100 from to make 900, then borrow 10 from to make 90, and give that 10 to the 2 to make 12! Math requires imagination. We don't want to have to write all of this out every time, so we use our imagination with the actual values and write 1 instead of 1,000, 9 instead of 900, etc.

Example 4.6. Joe's family ate out for $127.49. He had a coupon for 20% off, or $25.50. With the coupon, how much did Joe's family eat out for?

solution: In this problem, the story is that Joe used a 20% off coupon, which reduced the price of eating out. The key phrase is "20% off," which is a clue this is a subtraction problem. "With the coupon" tells us Joe used the coupon to reduce the price he had to pay by $25.50. A third clue is this problem is from Lesson 4 on subtraction. As with addition (See Ex. 3.6), we line up the decimal places. Then we subtract:

$$\begin{array}{r} \$127.49 \\ - \quad \underline{\$25.50} \\ \mathbf{\$101.99} \end{array} \qquad \text{Check by adding:} \qquad \begin{array}{r} \$101.99 \\ + \quad \underline{\$25.50} \\ \mathbf{\$127.49} \end{array}$$

Practice Set 4

The subscripted number next to the problem number references which lesson the problem is from.

1_4. 2+7=9 is a fact family. List three more fact families using these numbers, 1 addition and two subtraction facts.

2_4. 6-2=4 is a fact family. List three more fact families using these numbers, 1 subtraction and two addition facts.

3_4. Subtract the following pair of whole numbers. $\begin{array}{r} 31 \\ \underline{-18} \end{array}$

4_4. Subtract the following pair of whole numbers. $\begin{array}{r} 5,755 \\ \underline{-\ 2,109} \end{array}$

5_4. Add and subtract. Simplify inside parentheses first. 6-(4-1)+2+7

6_4. Subtract the following pair of real numbers. $5.49-$2.99

7_4. Subtract the following pair of real numbers. $\begin{array}{r} 5,100.0 \\ \underline{-\ \ 3,175.5} \end{array}$

8_4. Normally, the shoes cost $150.00, but the price was reduced by $29.99. What is the reduced price for the shoes?

9_3. Add the following pair of whole numbers. $\begin{array}{r} 47 \\ \underline{+0} \end{array}$

10_3. Add the following whole numbers. 2+1+5+0+3

11_3. Add the following pair of real numbers. $\begin{array}{r} 31,475.2 \\ \underline{+\quad 8,767.9} \end{array}$

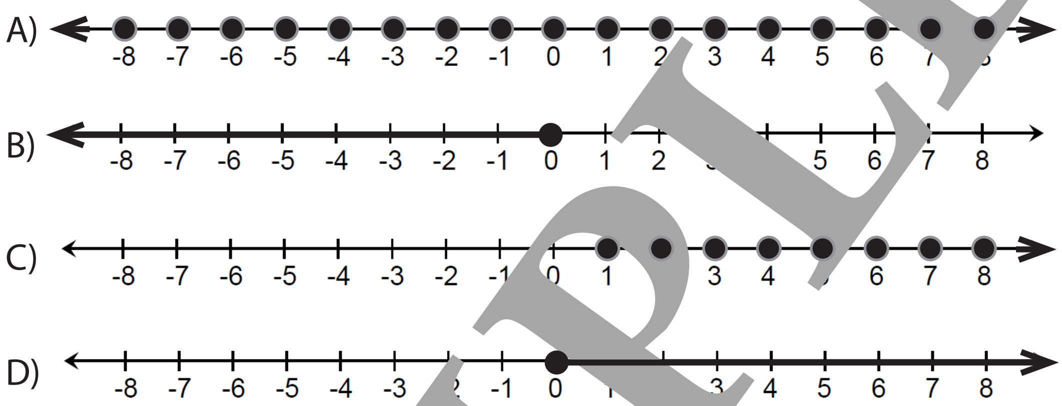
12_3. Add the following real numbers. 3.1+12.5+215.6

13_3. Add.

$$\begin{array}{r} \$39.95 \\ + \ \ \$8.29 \\ \hline \end{array}$$

14_3. The case of hunting ammo cost Braden \$199.95. Sales tax was \$7.99. How much did Braden pay altogether?

15_2. Which number line represents integers?



16_2. Compare 112 □ 121. Use a number line to help you.

17_2. What is the numerical value of the 7 in 370,000,005,000?

18_2. Use words to write the number in Problem 17.

19_1. In all Shormann Math courses, we define mathematics as the language of ______________ and a God-given tool for measuring and classifying pattern and shape.

20_1. Fill in the missing numbers in the following sequence.

18, 16, ____, ____, ____, 8...

Lesson 5 Maps and the Coordinate Plane; Ratio and Relating "this to that"

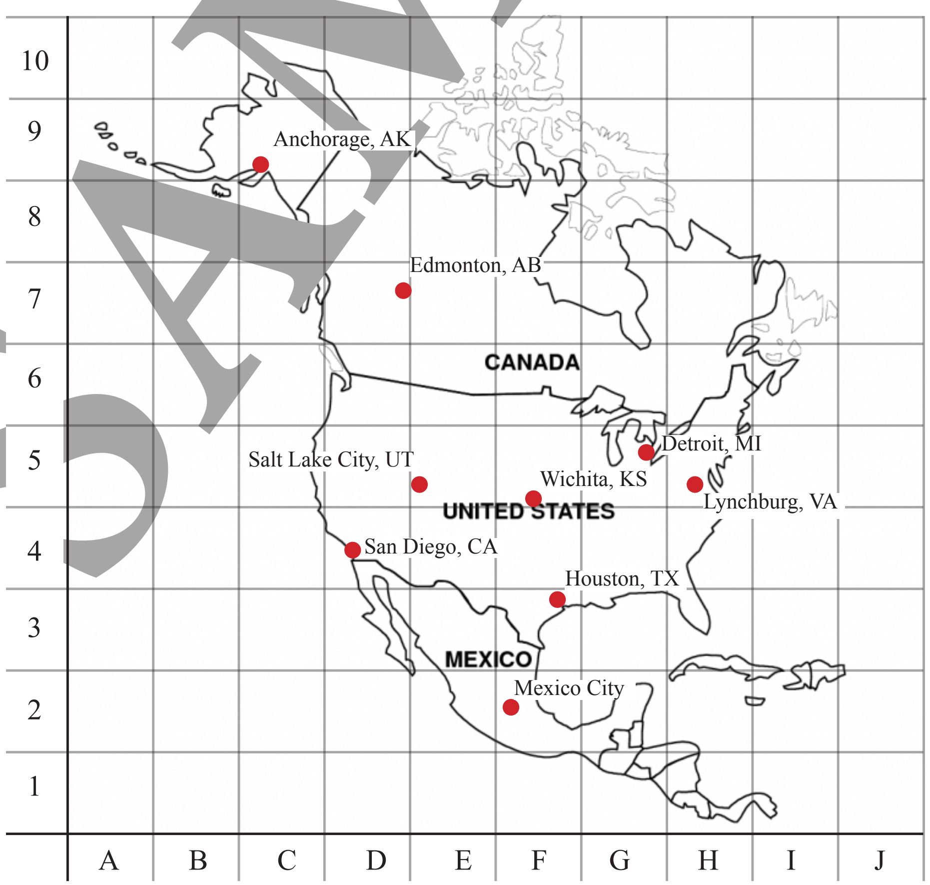
No New Rules

Definitions

ratio: The size of one thing relative to another. When you hear the word "ratio," think "fraction."
ordered pair: A pair of numbers, written in a specific order. On a coordinate plane, an ordered pair is used to identify the location of a point, and has the form (x,y). Also known as *coordinates*.
coordinate plane: Also called a *Cartesian coordinate system,* after Rene Descartes, it is a plane containing a horizontal, "x"-axis and vertical, "y"-axis.
origin: Where axes intersect on a coordinate plane, normally represented by the coordinates (0,0).

5A Maps and the Coordinate Plane

A coordinate plane is basically a way to identify a location on a flat, or plane surface. Maps are a type of coordinate plane. We'll use this very basic map of North America to identify locations of a few cities:

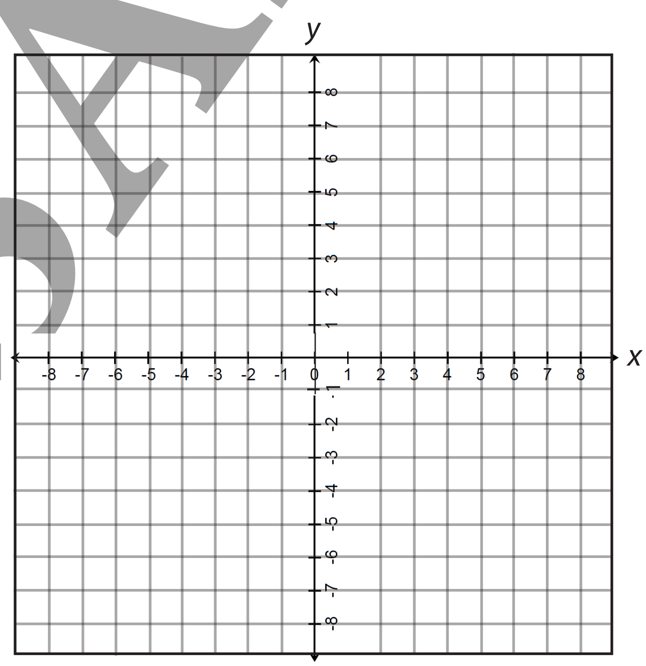


Notice the letters along the bottom of the map, and the numbers along the left side. To identify a location, let's use San Diego's location as an example. Normally, we start at the bottom left corner, and count letters over to where the red dot for San Diego is directly above, which is letter D. Next, we move up by numbers until we reach the dot, which is at location 4. So, we can estimate the location of San Diego as (D,4).

Example 5.1. Estimate the location of Houston, TX on the map.

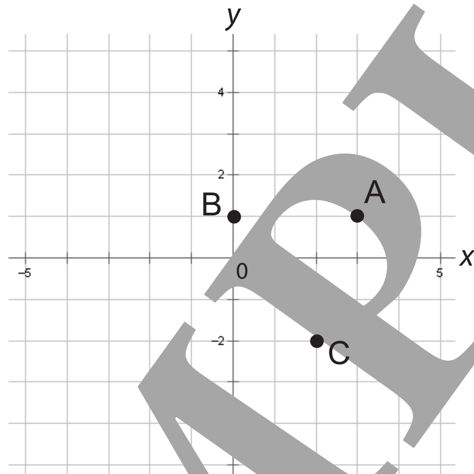
solution: Follow the same pattern for finding San Diego's location to estimate Houston's location. Move to the right until the red dot for Houston is directly above, which is letter F. Next, move up by numbers until we reach the dot, which is at location 3. So, we can estimate the location of Houston as **(F,3)**.

In math, we use a Cartesian coordinate system, named after Rene Descartes (1596-1650). In Ex. 4.1, the map coordinates provide an estimate of a point's location. For example, we know Houston is inside the box composed that is "F" wide and "3" tall. With a Cartesian coordinate system, we are still estimating positions, but the goal is to make a much more accurate estimate. Observe the following Cartesian coordinate system. It is made from two number lines (Lesson 2), one perpendicular to the other.



Try to draw your own Cartesian coordinate system. You don't need the grid lines, just draw two perpendicular number lines and label them *x* and *y*. We call a number line an *axis* in a coordinate system, so our horizontal number line is the "x-axis," and the vertical number line is the "y-axis." Locate "0" for both at their intersection. We call the intersection the *origin*.

Example 5.2. Identify the locations of points A, B and C on the following Cartesian coordinate plane. Write answers as an ordered pair, (x,y).



solution: ALWAYS start at the origin, identified by the "0" in the middle of the grid. Note on our map (Ex. 5.1), the origin was at the bottom-left, which is normal for maps. Next, follow a similar procedure for identifying the location of each point. For A, we move to the right 3 "tick marks" on the number line, then move up 1 to point A. Therefore, **A is located at (x,y)=(3,1)**.

For B, again, start at the origin. This time, we don't have to move left or right, so we move none to the left or right, and none is 0! Just move up 1 from the origin and you are at point B. Since we didn't move left or right along the x-axis, we write that **B is located at (x,y)=(0,1)**.

For C, start at the origin, then move to the right 2, then down 2. Note that "down" is in the negative y-direction. Therefore, **C is located at (x,y)=(2,-2)**.

Note how the *x* and *y* coordinates are different, but related. From the origin, you move the *x*-distance first. For the *y*-distance, you don't just start over at the origin. You start where the *x*-distance ends, and move up or down from there. You don't just ignore the *x*-distance. In fact, *y* depends on *x*! If you can remember that *y* depends on *x*, then identifying coordinates will be easy!

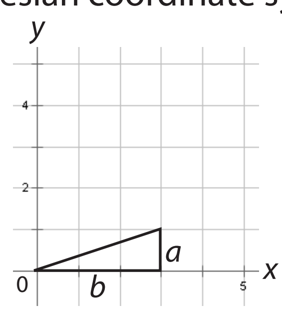
5B Ratio and Relating "This to That"

As stated in the definitions, ratio is about comparing things, usually how big they are. In our minds we like to just think about one thing, not two or more things. For example, have you ever seen a toddler walk into the path of a swing? The toddler is only thinking about one thing, that the swing is gone. They are not thinking about two things, like 1) the swing is gone and 2) the swing may return. And they are especially not thinking about how the position of the swing depends on other things, like time, gravity, etc. More simply, a toddler is not thinking about how *this compares to that*! They have no clue about the relationship between swing position and time.

In Lesson 4 we discussed how groups of numbers called fact families are related. And we discussed how it makes sense we would find relationships between numbers because math is created by God, who is relational. When things are related, we are able to compare them. We can compare this to that, which is what ratio is about.

We most often think of the word *fraction* when describing ratio. For example, if we are comparing two numbers, let's call them a and b, we might their ratio as a:b or a/b. In the following example, we will use what you learned in part A to create a ratio.

Example 5.3. Create a ratio, a/b, for the two sides of the triangle drawn on the graph. Use the Cartesian coordinate system to measure a and b.



solution: For the triangle, use the grid marks to see the triangle has a height of a=1, and the bottom side has a width of b=3. Therefore, the ratio **a/b = 1/3**.

That may seem super easy to you to look at a triangle and write down the ratio of the sides. I hope it was easy! Thinking about how this relates to that is really important in math, and in life! Comparing how one thing depends on others is at the heart of a topic called *analytical geometry* (Lesson 1). That's a really big phrase, and we are not concerned about it too much right now. But, we can learn some simple things about it.

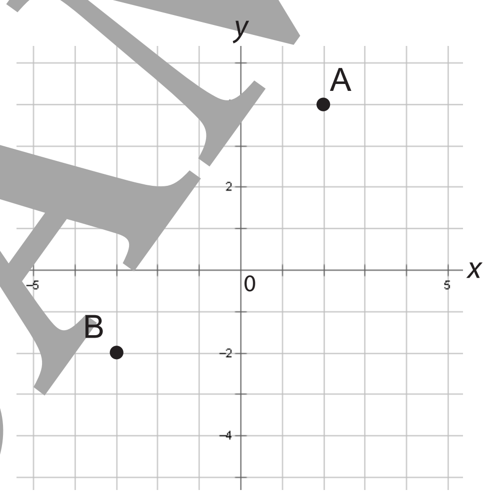
Practice Set 5

The subscripted number next to the problem number references which lesson the problem is from.

1_5. Use the map in Lesson 5A to estimate the location of Edmonton, AB in Canada.

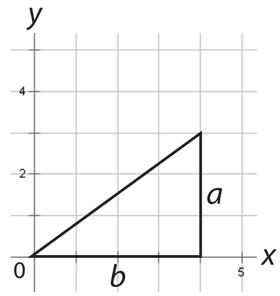
2_5. Use the map in Lesson 5A to estimate the location of Wichita, KS.

3_5. Identify the location of point A on the following Cartesian coordinate plane. Write answer as an ordered pair, (x,y). You will find point B in Problem 4.



4_5. Identify the location of point B on the graph in Problem 3. Write the answer as an ordered pair, (x,y).

5_5. Create a ratio, a/b, for the two sides of the triangle drawn on the graph. Use the Cartesian coordinate system to measure a and b.



6_5. According to the Shormann Math definition of ratio, when you hear the word "ratio," think "____________."

7_4. 1+2=3 is a fact family. List three more fact families using these numbers, 1 addition and two subtraction facts.

8_4. Subtract the following pair of whole numbers.

$$\begin{array}{r} 35 \\ \underline{-\ 16} \end{array}$$

9_4. Add and subtract. Simplify inside parentheses first. 5-2+(4-3)+7

10_4. Subtract the following pairs of real numbers.

$$\begin{array}{r} 1{,}000.6 \\ \underline{-\quad 501.5} \end{array}$$

11_4. Normally, the fishing reel cost $174.99, but the price was reduced by 25%, which is about $43.75 less. What is the reduced price for the fishing reel?

12_3. Add the following pairs of whole numbers.

$$\begin{array}{r} 44 \\ \underline{+29} \end{array}$$

13_3. Add the following whole numbers. 2,129+58+597

14_3. Add the following pairs of real numbers. 1.0004+5.1468

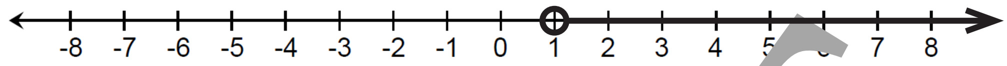
15_3. Add the following real numbers. 2.2+11.1+777.7

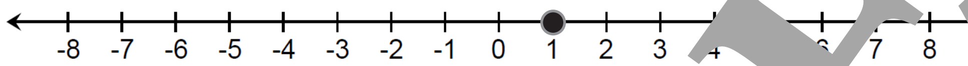
16_3. Add.

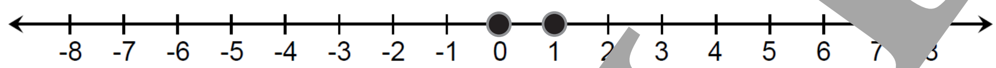
$$\begin{array}{r} \$42.99 \\ \underline{+\ \ \$18.00} \end{array}$$

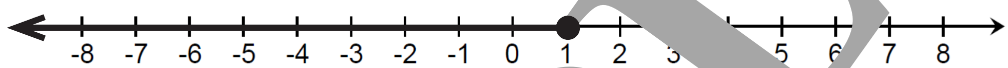
17_3. The farmer's market had watermelons for $8.50 and potatoes for $18.00 for a 30 lb sack. If Esther bought one watermelon and one sack of potatoes, how much did she owe altogether?

18_2. c<2 means "counting numbers less than 2." Which number line represents c<2?

A) 

B) 

C) 

D) 

19_2. Use words to write the number 20305050.

20_1. The Bible, in John 14:6, says that God is __________.

Lesson 6 What is Algebra? Addition and Subtraction Evaluations

A fisherman adds another Alaskan cod to a cartload of fish. In algebra, we can use symbols like "x" to represent the unknown quantity fish in a cart.

Rules

Multiplicative identity: $a \cdot 1 = a$. Any number times 1 equals that number.
Multiplication property of zero: $a \cdot 0 = 0$. Any number times 0 equals 0.

Definitions

algebra: A math tool where numbers are represented as letters and combined according to the rules of arithmetic, often to solve for an unknown value.

variable: In mathematics, a variable is a letter we use to represent any number. The numbers can vary, which is why we use the word *variable* to describe the letters.

evaluate: To find the value of an operation written in terms of *variables*, or letters, by assigning numbers to the letters and solving.

6A What is Algebra?

Algebra was discovered over 1,000 years ago by Mohammed ibn Musa al-Khowarizmi, who lived near modern-day Baghdad, Iraq. His book, *Hisab* ***al-jabr*** *w'al-muqabalah*, means "science of transposition and cancellation". An alternative meaning is the Book on Calculation by Completion and Balancing. That all may sound like a mouthful, because it is! We won't be doing any "al-jabr" in Lesson 6, but we will do some things that will help make algebra easier when you get there!

For over 1,000 years, people have been using algebra to help them make the unknown, known. In the photo at the top of Lesson 6, a cart contains an unknown quantity of Alaskan cod and halibut. Since we don't know exactly how many fish are in the cart, we could use a *variable*, or letter to represent the unknown quantity. We might write, "there are *x* number of fish in the cart." In the example above, we use "x" to represent an unknown quantity of fish. But, an "x" is just a symbol, meaning we could also use *a, b, c, d*, etc. We could even just sketch a bag of rocks and use that for our symbol, like this:

 = a bag of rocks of unknown quantity

Thinking of the bag of rocks, if  = x, then  = 2x,  = 3x, etc. Here, we use "x" to mean "one bag of rocks", 2x means "two bags of rocks", 3x means "three bags of rocks", etc. Notice we didn't write  = 1x, because 1x=x, according to the *multiplicative identity* in the Rules.

Did you notice we already used variables several times? The first time was in Ex. 2.4. We used variables to describe addition properties and identities in Lesson 3. We looked at the ratio *a/b* in Lesson 5. It's almost impossible NOT to use variables in math, because they are really helpful!

Example 6.1 If  = 5, what does  equal?

solution: There are 2 bags with 5 rocks each, so we multiply to get 2·5 = **10**. We could also add 5+5=**10**.

Example 6.2 If  = *b*, what does  equal?

solution: Follow the pattern in Ex. 6.1. The only difference is that 5 is now *b*. In Ex. 6.1, we multiplied 2(5)=10. Here, we just multiply 2(b) to get **2b**.

A key to understanding algebra problems is to use your imagination. If 2b is confusing, then think to yourself "2 times an unknown is similar to 2 times a known."

6B Evaluating Expressions

By *evaluate*, we mean "find the value." By *expression*, we mean something that looks like an arithmetic problem, except it has letters, or variables. ALL evaluate problems in this course will give you numbers to assign to the variables. **You replace the variables with those numbers, and then solve**. And that is all you do!

In math, *evaluate* means "find the value." Also, you might think of the expressions like machines that make things. Your expression is the machine, and you *input* values to it and it gives you an *ouput*, which is your answer.

Example 6.3. Evaluate each expression when a=1 and b = 2.

a) $a + b$ b) $b - a$

solution: Replace a with 1 and b with 2, and solve:

a) $a + b = 1 + 2 =$ **3**

b) $b - a =2 - 1 =$ **1**

Example 6.4 Evaluate each expression when x = 1 and y = 7.

a) $x + y$ b) $2y - x$

solution: Did you notice a=1 in Ex. 6.3, but x=1 in Ex. 6.4? How can two different variables both equal 1? Well, IT DOES NOT MATTER, they're *variables*! We can assign whatever number we want to them. The numbers can be the same, or they can be different. There is not some rule you are forgetting that says a and only a can equal 2. That's not how variables work. *On evaluate problems, your only job is to replace the variables with numbers and solve*. Now, replace x with 1 and y with 7 and evaluate each expression:

a) x + y = 1 + 7 = **8**

b) 2y - x =2•7 - 1 = 14-1=**13** Note that 2y also equals 1y+1y=y+y=7+7=14

Practice Set 6

The subscripted number next to the problem number references which lesson the problem is from.

1_6. If  = 3, what does  equal?

2_6. If  = *x*, what does  equal?

3_6. Evaluate $c + d$ when c=4 and $d = 2$.

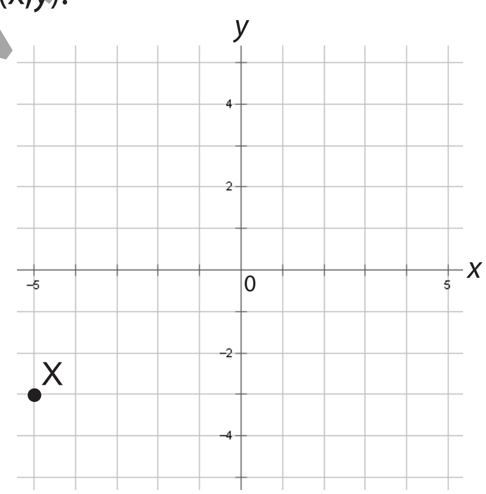
4_6. Evaluate $c - d$ when c=4 and $d = 2$.

5_6. Evaluate $2x + y$ when x=2 and $y = 3$.

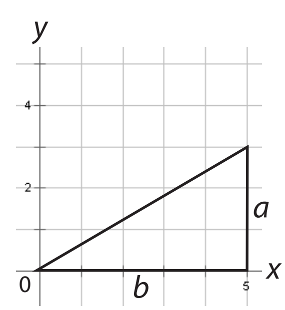
6_6. Evaluate $2x - y$ when x=2 and $y = 3$.

7_5. Use the map in Lesson 5A to estimate the location of Lynchburg, VA.

8_5. Identify the locations of point X on the following Cartesian coordinate plane. Write your answer as an ordered pair, (x,y).



9_5. Create a ratio, a/b, for the two sides of the triangle drawn on the graph. Use the Cartesian coordinate system to measure a and b.



10_4. 6-5=1 is a fact family. List three more fact families using these numbers, 1 subtraction and two addition facts.

11_4. Subtract the following pair of whole numbers.

$$\begin{array}{r} 7{,}400 \\ \underline{-\ 5{,}280} \end{array}$$

12_4. Add and subtract. Simplify inside parentheses first. 4-(5-2)+7+8+3

13_4. Subtract the following pairs of real numbers. \$8.50-\$2.99

14_4. The surf foiling boards normally sold for \$1,099.00, but were on sale for 70% off, or \$769.30 off. What was the sale price for the boards?

15_3. Add the following pair of whole numbers.

$$\begin{array}{r} 2{,}445 \\ \underline{+8{,}657} \end{array}$$

16_3. Add the following whole numbers. 1+5+4+2+6

17_3. Add.

$$\begin{array}{r} \$69.99 \\ \underline{+\ \$23.49} \end{array}$$

18_3. Which of the following is the *additive identity*?

A) If $a + b = c$, then $b + a = c$

B) $(a + b) + c = a + (b + c)$

C) $a+0 = a$

D) $a \cdot 1=a$.

19_3. Which of the following is the cent symbol in the U.S. System?

A) **¢**

B) **\$**

C) **+**

D) **∊**

20_2. Compare 5 □ -1. Use a number line to help you.

Lesson 7 Missing Numbers in Addition and Subtraction

Review: Lessons 2, 3, 4, 6

Rules

For **addition with a missing number**, remember these rules:

1) If the sum is missing, use addition to solve.
2) If either addend is missing, use subtraction to solve.

For **subtraction with a missing number**, remember two rules:

1) If the minuend is missing, use addition to solve.
2) If the subtrahend or difference is missing, use subtraction to solve.

Definition

equation: A statement that uses the "=" symbol (Lesson 2) to show that values on either side of the "=" symbol are equal.

7A Missing Numbers in Addition

Lesson 7 combines what you learned in previous lessons to find the value of missing numbers found in equations. We have worked with equations some already, especially with fact families, beginning in Lesson 4. We included a definition for *equation* above.

The main things we are combining in Lesson 7 are fact families (Lessons 3 and 4) and variables (Lesson 6). You will also want to review basic arithmetic definitions found in Lessons 3 and 4.

Example 7.1 Find the missing number in each addition problem.

a) $8 + 4 = x$ b) $6 + a = 10$ c) $b + 4 = 30$

solution: a) This is the easiest type of missing number problem, since both numbers are on one side of the equation. Simply add 8+4=12. First though, standard form is to have the variable on the left side of the equals sign. It doesn't matter, 8+4=x is the same thing as x=8+4. But, we are using letter x for the missing number, and it makes more sense to say "x equals twelve," than "twelve equals x." We will always arrange missing number problems so the variable is on the left:

8+4=x

x=8+4

x=12

b) There is more than one way to solve these. One way is to think to yourself "6 plus what equals 10?" You probably know 6+4=10, so ***a*=4**.

Another way to solve it is to rearrange the three numbers into a subtraction fact. If 6+a=10, then we can write the subtraction fact

a=10-6

***a*=4**

c) Like b), we can rearrange into a subtraction fact:

b=30-4

b=26 Check with addition: 26+4=30

7B Missing Numbers in Subtraction

Like addition, finding missing numbers in subtraction is a matter of rearranging into an appropriate fact family and solving.

Example 7.2 Find the missing number in each subtraction problem.

a) 10-2=x b) *N* - 1 = 6 c) 17 - *b* = 14

solution: a) Like Ex. 7.1a, this one is easy, just put the x first:

x=10-2

x=8 Again, it is not "wrong" to write 8=x, it's just not standard form. We will always try to write answers in standard form.

b)If you thought "7 - 1 = 6, so **N=7**," you are correct! Check your work using fact families, because if 7, 1 and 6 are truly a fact family, then you can make other facts, like 1+6=7.

Now, I also want you to see how you can rearrange the original problem using the Lesson 7 Rules. Since the minuend is missing, our rule is to rearrange this into an addition problem:

N-1= 6

N=1+6

N = 7

c) This time, the subtrahend is missing, so we apply our Lesson 7 Rule and rearrange this into another subtraction fact. Put the larger number first:

$17 - b = 14$

$b = 17-14$

b = 3 Check your work with addition: $3+14 = 17$.

Practice Set 7

The subscripted number next to the problem number references which lesson the problem is from.

1_{7}. Find x. $3 + 10 = x$

2_{7}. Find t. $6 + t = 11$

3_{7}. Find k. $k+9=15$

4_{7}. $1 + b = 10$

5_{7}. Find c. $c+13=20$

6_{7}. Find y. $14-4=y$

7_{7}. Find M. $M - 2 = 11$

8_{7}. Find f. $21 - f = 17$

9_{7}. Find z. $35 - z = 22$

10_{6}. If  = 10, what does  equal?

11_{6}. Evaluate $c + d$ when $c=4$ and $d = 5$.

12_{6}. Evaluate $2x + y$ when $x=5$ and $y = 21$.

13_{5}. Use the map in Lesson 5A to estimate the location of Detroit, MI.

14_{5}. Identify the location of point D on the following Cartesian coordinate plane. Write answer as an ordered pair, (x,y).

15_{5}. Create a ratio, a/b, for the two sides of the triangle drawn on the graph. Use the Cartesian coordinate system to measure a and b.

16_{4}. 8+2=10 is a fact family. List three more fact families using these numbers, 1 addition and two subtraction facts.

17_4. Subtract. $\begin{array}{r} 31 \\ \underline{-18} \end{array}$

18_3. Add. $\begin{array}{r} 31 \\ \underline{+18} \end{array}$

19_3. Casey's family ate out for \$108.45. He added a tip of \$17.35. How much did Casey pay altogether?

20_1. Mathematics is the language of science and a God-given ________for measuring and classifying pattern and shape.

Lesson 8 Multiplication with Whole Numbers; Making Line Graphs from Ordered Pairs

Review: Lessons 3, 5, 6

Rules

Commutative property for multiplication: If $ab = c$, then $ba = c$

Order does not matter. Example: $2 \cdot 3 = 6$, and $3 \cdot 2 = 6$

Associative property for multiplication: $(a \cdot b)c = a(b \cdot c)$

When you have more than two numbers to multiply, the way you group them in pairs does not matter. Example: $(2 \cdot 3) \cdot 4 = (6) \cdot 4 = 24$, and $2 \cdot (3 \cdot 4) = 2 \cdot (12) = 24$

× = multiplication sign Used to indicate multiplication of two numbers. Alternative symbols include • and (). For example, 3×4, 3•4, 3(4), and (3)(4) all mean "3 times 4."

Definitions

factor: A number being multiplied.

product: The result of multiplication. For example, *factor × factor = product.*

point: That which has no part. Its location is represented by a dot. • ← a point

line: A widthless length. Its location is represented on paper by using a pencil and straight edge. Arrow tips show it continues forever in that direction.

8A Multiplication with Whole Numbers

Multiplication is really just a way to quickly add equal groups, or "bags of rocks" like we did in Lesson 6. Here, we will review multiplying 2 and 3-digit whole numbers. Note also that, like addition and subtraction create fact families, multiplication and division create fact families. We will review division in Lesson 9. The commutative property for multiplication in the Rules helps us identify two multiplication facts, since order doesn't matter.

Example 8.1 Multiply.

$$\begin{array}{r} 32 \\ \underline{\times\ 18} \end{array}$$

solution: When we multiply, we multiply one place value at a time. First, we multiply 32 by the 8 in 18. The method reminds us of addition. We multiply the 8 by 2 to get 16, write down the 6 below the line, carry the 1, then multiply 8

$$\begin{array}{r} {}^{1}32 \\ \underline{\times\ 18} \\ 256 \\ \underline{+\ 320} \\ \mathbf{576} \end{array}$$

by 3 to get 24 and add the 1 we carried. We write 25 below the line to show $32\times8=256$.

Then on a new line, we multiply the 32 by 1, which is the multiplicative identity (Lesson 6 Rules) and therefore just equals 32. Note how we put a placeholder 0 in the ones place, because the 1 in 18 is in the tens place, so we are really multiply 32 by 10. Understanding place value is a big part of correctly multiplying larger numbers.

Finally, we add the two products together, $256+320=576$. To recap, we 1) multiplied 8 by 32, 2) multiplied 1 by 32, adding a placeholder zero, and 3) added the products.

Example 8.2 Find the product. $\begin{array}{r} 320 \\ \underline{\times\, 607} \end{array}$

solution: The phrase "find the product" is equivalent to "multiply." First, we multiply $320\times7=2{,}240$. Next, we have a 0 in the tens place, and from the multiplication property of zero (Lesson 6), we know $320\times0=0$. There are different ways to write the product, so we wrote a placeholder zero, and another zero for the tens place. For the third row, we multiply $320\times6=1{,}920$. The 6 is in the hundreds place, so we need two placeholder 0's, which gives 192,000. Finally, we add the three products to get 194,240.

$$\begin{array}{r} 320 \\ \underline{\times\, 607} \\ 2{,}240 \\ 00 \\ \underline{+192{,}000} \\ \mathbf{194{,}240} \end{array}$$

Example 8.3 Find the total cost for 3 dozen apples that cost 70¢ each, and then convert to dollars.

solution: Review Lesson 3 Rules on money symbols. Hopefully you know 1 dozen = 12, but if not, now you do! Skip count or multiply to get 36 apples, then multiply by 70. It does not matter which number is on top, but we will normally put the larger number on top:

$$\begin{array}{r} 70 \\ \underline{\times\ 36} \\ 420 \\ +\underline{2{,}100} \\ 2{,}520¢ \end{array}$$

To convert to dollars, think about how 100¢=$1.00. We can convert from cents to dollars by moving the

decimal place to the left two places, like this:
2,520 ¢ = **$25.20**

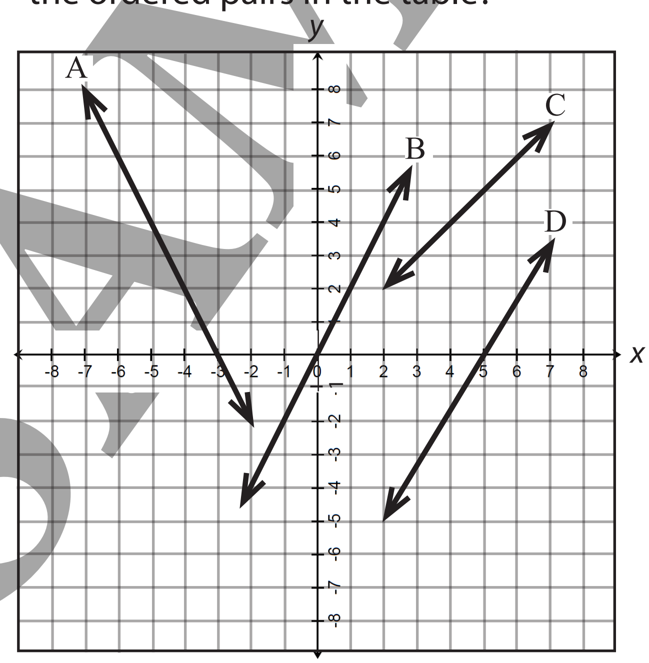
8B Making Line Graphs from Ordered Pairs

In Lesson 5, you learned about plotting points on a Cartesian coordinate system, or *graph* for short. We called the points, ordered pairs. We use a dot to represent the location of the point. We define both *point* and *line* in this lesson.

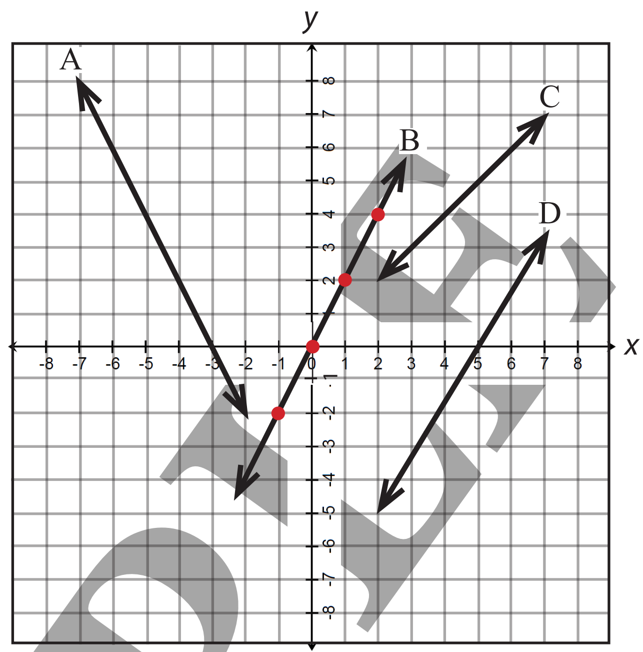
A big part of math is identifying patterns. In Lesson 8B, we will plot several points listed in a table. All these tables will have points that, when we plot them, create a linear pattern. By "linear pattern," we mean they follow a pattern that, if you use your imagination, looks like a line! Identifying linear patterns is a really important part of math, especially when applied to science, engineering and other subjects.

Example 8.4 Which of the following lines best fits the pattern created by the ordered pairs in the table?

x	y
-1	-2
0	0
1	2
2	4



solution: In Ex. 5.2, you were given point locations and asked to find the ordered pair. Here, you are given a table of ordered pairs, so just do Ex. 5.2 in reverse and find their locations. The first point is (-1,-2), followed by (0,0), (1,2) and (2,4). You can create a new graph, or just plot them on the graph above and you will see that **line B** is correct. Here is the same graph with the points plotted, making it obvious **line B** is correct.



Practice Set 8

The subscripted number next to the problem number references which lesson the problem is from.

1_8. Multiply. $\begin{array}{r} 27 \\ \underline{\times \quad 5} \end{array}$

2_8. Find the product of 32 and 19. In other words, multiply. $\begin{array}{r} 32 \\ \underline{\times 19} \end{array}$

3_8. Multiply 70 and 14.

4_8. Multiply. $\begin{array}{r} 215 \\ \underline{\times 473} \end{array}$

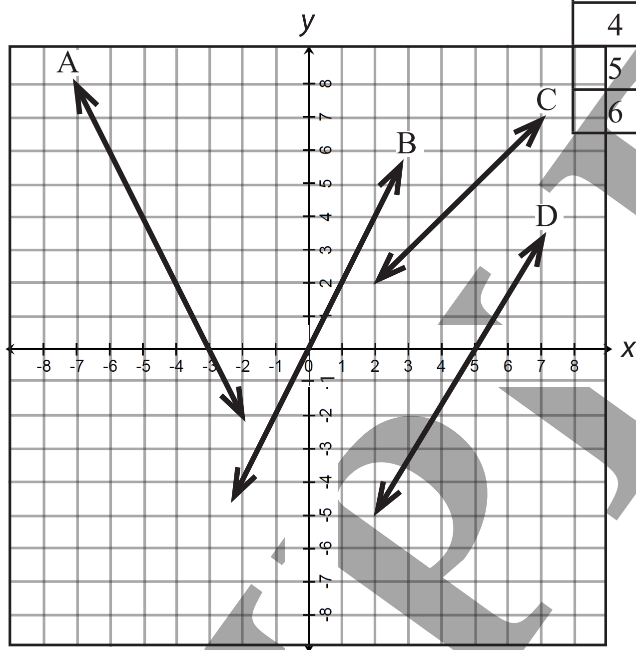
5_8. Find the product. $\begin{array}{r} 400 \\ \underline{\times \quad 29} \end{array}$

6_8. Multiply. $\begin{array}{r} 606 \\ \underline{\times \quad 199} \end{array}$

7_8. Find the total cost for one gross of limes that cost 15¢ each, and then convert to dollars. Note: 1 gross = 144

8_8. Which of the following lines best fits the pattern created by the ordered pairs in the table?

x	y
3	3
4	4
5	5
6	6



9_7. Find *d*. $3 + d = 14$

10_7. Find *d*. $d + 5 = 9$

11_7. Find *M*. $M - 5 = 8$

12_7. Find *k*. $23 - k = 18$

13_6. If  = *a*, what does  equal?

14_6. Evaluate *a-b* when *a*=10 and *b*=3.

15_6. Evaluate *x-2y* when *x*=9 and *y*=1.

16_5. Use the map in Lesson 5A to estimate the location of Salt Lake City, UT.

17_4. Subtract.
$$\begin{array}{r} 488 \\ \underline{-\ 189} \end{array}$$

18_3. Add.
$$\begin{array}{r} 4{,}351 \\ \underline{+2{,}869} \end{array}$$

19_3. Add the following real numbers. 2.2+18.9+84.9

20_2. What do we call numbers less than zero?
A) natural numbers B) whole numbers
C) negative numbers D) positive numbers

Lesson 9 Division with Whole Numbers; Order of Operations with × and ÷

Review: Lessons 3, 4, 8

Rules

÷ = **division sign** Used to indicate division of two numbers. Alternative symbol is /. For example, 12÷4 and 12/4 can both mean "12 divided by 4."

Definitions

dividend: The number divided by the divisor. Usually the larger number.
divisor: The number dividing the dividend.
quotient: The result of division. For example, *dividend ÷ divisor = quotient*

9A Division with Whole Numbers

Fact families help us connect multiplication and division. Let's do a couple of examples before diving deeper into division.

Example 9.1 Write three more fact families for 6÷2=3.

solution: With fact families, we can see that multiplication "undoes" division, and vice versa. For example, we are given that 6 divided by 2 equals 3. Using our Lesson 9 definitions, we can "undo" the division problem by multiplying the quotient of 3 by the divisor of 2. And, since order doesn't matter in multiplication (Lesson 8 Rules), we can create one more multiplication fact. Our second division fact is made by switching the quotient and divisor. Therefore, our three facts are

3×2=6 2×3=6 6÷3=2

Example 9.2 Write three more fact families for 2×4=8.

solution: We can write another multiplication fact by switching the 2 and 4. For our two division facts, we use the 8 as the dividend (usually the largest number), and switch the divisor and quotient. Therefore, we get

4×2=8 8÷2=4 8÷4=2

Next, let's review long division. First though, review the Lesson 9 Definitions for the names of the parts of a division problem. When learning arithmetic, addition is almost always learned first, and division is learned last. The reason for that is because division is normally more difficult. But, you can make it easier by memorizing rules and definitions, and connecting it to multiplication via fact families.

When we divide larger numbers, we use a long division symbol to help us set up the problem. The dividend always goes inside the long division symbol. Here are three ways to identify where these parts are in a division problem:

$$\begin{array}{r} \textit{quotient} \\ \textit{divisor}\overline{\big)\textit{dividend}} \end{array} \qquad \frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient} \qquad \textit{dividend} \div \textit{divisor} = \textit{quotient}$$

For addition, subtraction and multiplication, we presented step-by-step procedures, or *algorithms*, to find results. Now, let's do the long division algorithm!

Example 9.3 Divide a) $235 \div 5$ b) $6\overline{)78}$
Check using multiplication.

solution: a) The larger number is the dividend. We can also look at the symbolic forms above to see the dividend ÷ divisor = 235 ÷ 5. Using long division to solve, this means the 5 goes on the outside left, and the 235 inside the long division symbol. Next,

solve
$$\begin{array}{r} \mathbf{47} \\ 5\overline{)235} \\ \underline{-20}\downarrow \\ 35 \\ \underline{-35} \\ 0 \end{array}$$
and check
$$\begin{array}{r} \overset{3}{4}7 \\ \underline{\times\, 5} \\ 235 \end{array}$$

The BEST way to understand long division is to watch the video lecture. Explaining in writing is laborious, but one thing we are doing is always thinking about the divisor. First we know the divisor is 5, so we look at the first digit in the dividend, the 2, and think "what times 5 equals 2?" There is no whole number that answers this question, so we add the next digit, the 3, and now we think "what times 5 equals 23?" Again, nothing answers that question, BUT, we can get close, since 5×4=20. So, we put a 4 above the 3, then subtract 23-20=3. Next, we bring down the 5 (that's what the arrow is showing) to get 35. We repeat the question, this time thinking "what times 5 equals 35?" Well 7×5 equals EXACTLY 35, so we put the 7 in the quotient, after the 4, then subtract and get 35-35=0. We call this result of zero the *remainder*. Sometimes the remainder does not equal zero, but **for Lesson 9, all division problems will have a remainder of zero**. To conclude, our quotient = **47**. To the right of the division algorithm, we checked our work using a multiplication fact.

b) This problem is already set up in long division format. Divide and check as in a):

Solve:

13
6)78
-6
18
-18
0

Check:

13
× 6
78

Example 9.4 Divide. a) 897 ÷ 23 b) 500 ÷ 25

solution: Rewrite in long division format, divide, and check:

a)

39
23)897
- 69
207
-207
0

Check:

23
× 39
207
+690
897

a) and b) both have 2-digit divisor. Two-digit divisors are more challenging because you are not as familiar with skip counting or using multiplication facts stored in your memory. But, we apply the division algorithm the same way, which means we start by recognizing 23 times nothing equals 8, so then we add the 9 and quickly you can see that 23 "goes into" 89, but by how much? This is where you could start getting lazy, but don't let yourself! Use a pencil and paper and do a couple of multiplications to see what works, like 23×3 and 23×4:

```
  1
 23    23
x 3   x 4
----  ----
 69    92
```

23×4 is too big, so we use 23×3 (we say 23 "goes into" 89, 3 times), put the 3 in the quotient and then subtract 89-69=20. 23 won't "go into" 20, so we bring down the 7 and get 207. Again, you can do some multiplications over to the side. Remember too, these problems are designed to have a remainder=0, so 23 times something MUST equal 207. One thing that helps sometimes is to think of an easy multiplication, like 23×10=230. That's too big, but look, it's not that much bigger than 207, so maybe 23×9 works. Do the math and you will see that 23×9=207.

b) This might be easier than a) because the numbers are more familiar and are multiples of 5. However, all the zeros can be confusing so be careful with your answer and check your work:

```
    20
25)500                  ¹25
  - 50↓                 × 20
    00        Check:      00
   - 0                  +500
     0                   500
```

25 times nothing equals 5, but 25×2=50, so write a 2 in the quotient and subtract 50-50=0. Bring down the 0. Since we have 0, you may think you are done, but think about it, all you have done so far is show that 25×2=50, so the quotient can't be 2. We can continue the algorithm with the 0's there, because 25×0 equals a number! It equals 0. We put a 0 in the quotient and, even if it seems strange, subtract 0-0=0. Therefore, the quotient is **20**, and we can check using multiplication.

9B Order of Operations with Multiplication and Division

We introduced PEMDAS in Lesson 4, another algorithm for solving problems. Here we introduce the M and D (E for exponents will be last). The letters are arranged to help you remember that we perform multiplication and division before addition and subtraction.

Example 9.5 Simplify the following.

a) 6(5+1)-(3÷3)+2
b) 18/2-5+3•2

solution: Apply PEMDAS, and do multiplication and division before addition and subtraction:

a) $6(5+1)-(3\div3)+2$	ALWAYS, "same ÷ same=1." Remember that, and you can avoid a lot of confusion! $3\div3=1$, $x\div x=1$, etc. Continue simplifying:
$6(6) - (1)+2 =$	multiply first
$36 - 1 + 2 =$ $35 + 2 =$ **37**	We didn't add parentheses on because by now you probably know to just add/subtract in pairs.
b) $18/2-5+3\cdot2$	Recall that "/" also means division, and "•" means multiplication. First, do $18/2=9$ and $3\cdot2=6$:
$9-5+6 =$ $4 + 6 =$ **10**	Reading left to right, we subtract $9-5=4$, and end by adding $4+6=10$.

Practice Set 9

The subscripted number next to the problem number references which lesson the problem is from.

1_9. Write three more fact families for $12\div3=4$.

2_9. Divide. $414\div3$

3_9. Divide. $648\div24$

4_9. Divide. $800\div16$

5_9. Simplify the following. $3(4-1)-(8\div2)+10$

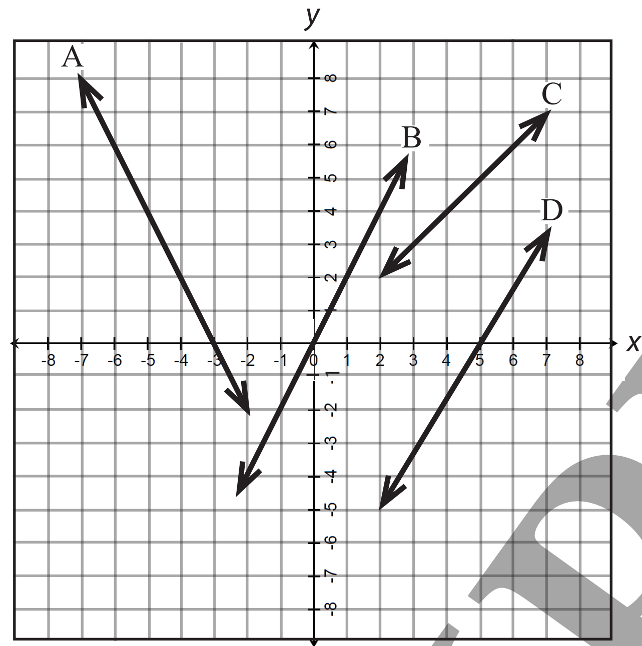
6_8. Multiply. $\begin{array}{r} 44 \\ \underline{\times\, 39} \end{array}$

7_8. Find the product. $\begin{array}{r} 408 \\ \underline{\times\, 581} \end{array}$

8_8. Find the total cost for 1 dozen potatoes that cost 89¢ each, and then convert to dollars.

9_8. Which of the following lines best fits the pattern created by the ordered pairs in the table?

x	y
-3	0
-4	2
-5	4
-6	6



10_7. Find *z*. $9+2 = z$

11_7. Find *y*. $4 + y = 12$

12_7. Find *x*. $x + 17 = 20$

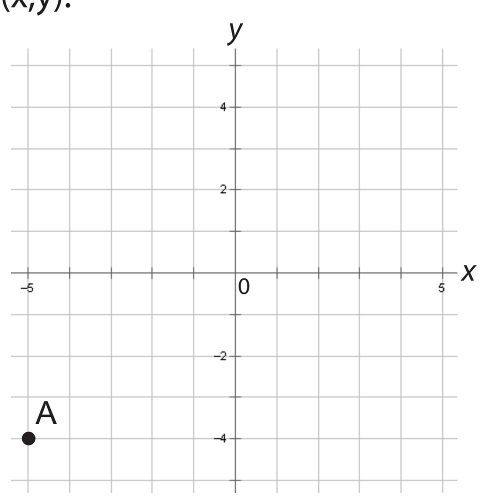
13_7. Find *N*. $N - 5 = 12$

14_7. Find *P*. $13 - P = 4$

15_6. If  = 12, what does  equal?

16_6. Evaluate $3a + b$ when $a=1$ and $b = 4$.

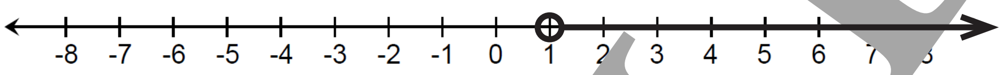
17_5. Identify the location of point A on the following Cartesian coordinate plane. Write answer as an ordered pair, (x,y).

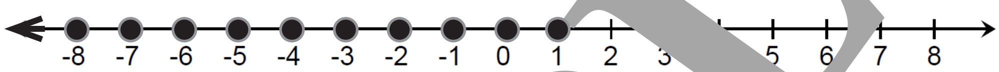


18_4. Simplify. 8-2+(11-7)+(3-3)+2

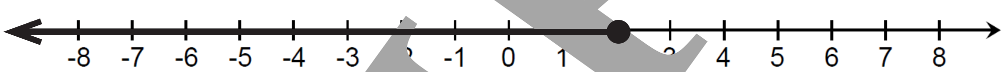
19_3. Add. 6.558+3.807

20_2. $\mathbb{Z} \leq 2$ means "integers less than or equal to 2." Which number line represents $\mathbb{Z} \leq 2$?

A)


B)


C)


D)


Lesson 10 Division with Non-zero Remainders; Division and Fractional Parts

Review: Lessons 5, 9

No new Rules

Definitions

fraction: Part of a whole. A numerical quantity that indicates division of one number by another. A ratio.
numerator: The top value in a fraction.
denominator: The bottom value in a fraction.
mixed number: A number that contains a whole number and a fraction, like $3\frac{5}{8}$.

10A Division with Non-zero Remainders

All division problems from Lesson 9 had remainders of zero. However, most division problems have "leftovers", meaning they have non-zero remainders. Sometimes, this is referred to as *uneven* division. When writing the quotients to uneven division problems, we can write the remainder as a whole number, or as a fraction. **Note:** when we use words to write a fraction, we will put a hyphen (-) between the numerator and denominator. For example, 1/4 = "one-fourth."

Example 10.1 Divide. Write the remainder as a whole number.
a) 9÷2 b)17÷5

solution: Perform the standard division algorithm, except, unlike in Lesson 9, you will get a non-zero remainder.

a) $\begin{array}{r} \textbf{4 R1} \\ 2\overline{)9} \\ \underline{-8} \\ 1 \end{array}$ The remainder is 1, so we write this as "R1." We can say the quotient equals "4 remainder 1."

b) $\begin{array}{r} \textbf{3 R2} \\ 5\overline{)17} \\ \underline{-15} \\ 2 \end{array}$ The remainder is 2, so we write this as "R2."

Example 10.2 Divide. Write the remainder as a fraction.

a) 29/8 b)50÷11

solution: To write the remainder as a fraction, the remainder is in the numerator and the divisor in the denominator:

a)Note that 29/8 means 29÷8:

$$\begin{array}{r} 3 \text{ R}5 \\ 8\overline{)29} \\ \underline{-24} \\ 5 \end{array} = \boxed{3\frac{5}{8}}$$

The remainder is 5, so we write this as the fraction, remainder/divisor = 5/8. We write the quotient as a *mixed number*, $\boxed{3\frac{5}{8}}$. We can say the quotient equals "three and five-eighths."

Check your work by multiplying 3(8), then adding the remainder of 5: 3(8)+5=24+5=29

$$\begin{array}{r} 4 \text{ R}6 \\ 11\overline{)50} \\ \underline{-\ 44} \\ 6 \end{array} = \boxed{4\frac{6}{11}}$$

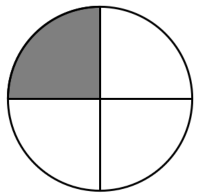
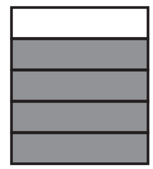
b) The remainder is 6, so we write this as $\boxed{4\frac{6}{11}}$. We can say this "four and six-elevenths."

Check your work by multiplying 4(11), then adding the remainder of 6: 4(11)+6=44+6=50.

10B Division and Fractional Parts

In Lesson 5, we said that when you hear the word ratio, think "fraction." We defined ratio as the size of one thing relative to another. With fractions, the "one thing" is usually the numerator, and the "another" is the denominator. We also think of fractions as a "part of a whole," where the part is the numerator, and the whole is the denominator.

Example 10.3 Look at the shapes shown. Each one is divided into an equal number of parts. What fraction is shaded?

a)  b) 

solution:

a) The circle is divided into 4 equal parts. One of the 4 is shaded, so we write the fraction shaded as $\boxed{\frac{1}{4}}$. We might say "One-fourth of the circle is shaded."

b) The rectangle is divided into 5 equal parts. 4 of the 5 are shaded, so we write the fraction shaded as $\boxed{\frac{4}{5}}$. We might say "four-fifths of the rectangle is shaded."

We also use "fractional language" to create division problems. For example, what is half of 10? You probably don't need to set up a long division problem to know that half of 10 equals 5. You just know that if you cut 10 in half, that would equal 5. Or, another example might be, if you had 10 apples, and you gave half of them to your brother, then you would give 5 of them to your brother. Let's use division to solve some more challenging fractional parts.

Example 10.4 Solve each of the following.

a) What number is 1/2 of 51? Write answer as a mixed number.

b) What number is 1/8 of 328?

solution: To solve for part of the whole, simply divide by the fraction's denominator. In a), divide 51 by 2, and in b) divide 328 by 8

a) You know half of 50 equals 25, so half of 51 must be slightly more:

```
  25 R1 = [25 1/2]
2)51
 -4
  11
-  10
    1
```

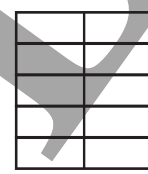
As in Ex. 10.2, check your work by multiplying 25(2), then adding the remainder of 1: 25(2)+1=50+1 = 51.

b) "1/8 of 328" means "divide 328 by 8":

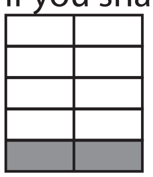
```
    41
 8)328
  -32
    08
 -   8
     0
```

You can check your work by multiplying 41×8=328

Example 10.5 Shade 1/5 of the figure shown.



solution: Copy the figure onto your paper and shade 1/5 of it. The shape has 10 equal parts, but observe carefully to see there are 5 rows. Therefore, if you shade one of the five rows, or 2 of the 10 parts, you shade 1/5 of it:



One conclusion to draw from this is that $\frac{1}{5} = \frac{2}{10}$. We call these equivalent fractions, and will learn more about them in later lessons.

Practice Set 10

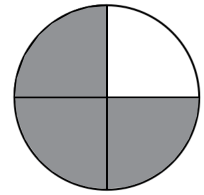
The subscripted number next to the problem number references which lesson the problem is from.

1_{10}. Divide. Write the remainder as a whole number. 14÷3

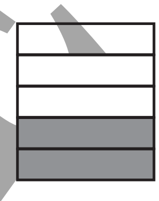
2_{10}. Divide. Write the remainder as a fraction. 31/7

3_{10}. Divide. Write the remainder as a fraction. 43÷12

4_{10}. The following shape is divided into an equal number of parts. What fraction is shaded?

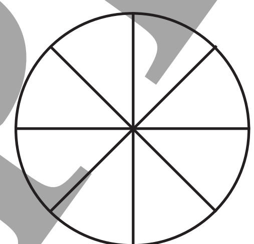


5_{10}. The following shape is divided into an equal number of parts. What fraction is shaded?



6_{10}. What number is 1/3 of 40? Write answer as a mixed number.

7_{10}. Shade 1/4 of the figure shown.



8_{9}. Write three more fact families for $3 \times 5 = 15$.

9_{9}. Divide. $4\overline{)116}$

10_{9}. Divide. $836 \div 44$

11_{9}. Simplify. $2 \cdot 3 - 24 \div 8 + 5$

12_{8}. Multiply. $\begin{array}{r} 21 \\ \underline{\times\ 46} \end{array}$

13_{8}. Find the product. $\begin{array}{r} 213 \\ \underline{\times\ 500} \end{array}$

14_{7}. Find t. $4 + t = 13$

15_{7}. Find H. $H - 5 = 3$

16_{6}. Evaluate $2y - 2x$ when $x=1$ and $y = 5$.

17_5. Create a ratio, a/b, for the two sides of the triangle drawn on the graph. Use the Cartesian coordinate system to measure a and b.

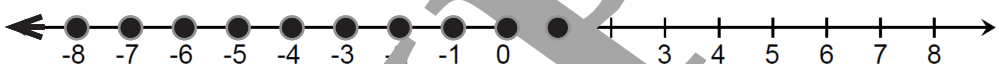
18_4. 8-2=6 is a fact family. List three more fact families using these numbers, 1 subtraction and two addition facts.

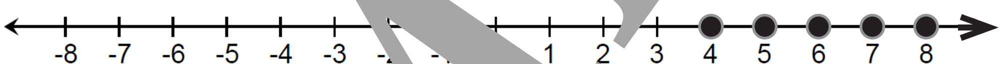
19_3. Add.

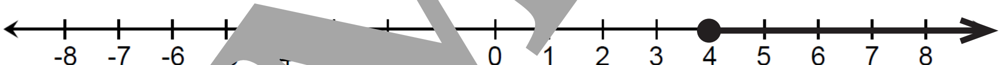
$$\begin{array}{r} 12{,}599.2 \\ +\quad 725.8 \\ \hline \end{array}$$

20_2. n>3 means "real numbers greater than 3." Which number line represents n>3?

A)


B)


C)


D)


Lesson 11 Rounding Whole Numbers and Decimals; Divisibility

Review: Lessons 1, 2, 9, 10

Rules

For rounding any number: Circle the place value (Lesson 2) you want to round to. Look at the digit to the right. If it is 5 or greater, round the circled digit up 1. Otherwise, leave it as is. Remove the digits to the right of the circled digit and replace them with zeros.

Rules for Divisibility:

A number is divisible by

2	if the last digit is even
3	if the sum of the digits is divisible by 3
4	if the last two digits are divisible by 4
5	if the last digit is 0 or 5
6	if the number is divisible by 2 and 3
7	If you can double the last digit and subtract it from the number formed by the remaining digits, and the new number is divisible by 7, then the original number is also divisible by 7
8	if the last 3 digits are divisible by 8 (Note: most numbers, if divisible by 2 and 4, are also divisible by 8)
9	if the sum of the digits is divisible by 9
10	if the last digit is 0
11	if you can subtract the last digit from the number formed by the remaining digits, and the new number is divisible by 11, then the original number is also divisible by 11
12	if the number is divisible by 3 and 4

Definitions

rounding up: If the Lesson 11 Rule tells us to "round up," then that means we need to add 1 to the circled digit.

rounding down: If the Lesson 11 Rule tells us to leave the circled digit as-is, we sometimes describe this as *rounding down*.

divisibility: The ability of a whole number to divide another whole number, yielding a remainder of zero.

even numbers: A sequence of whole numbers that, except for 0, are divisible by 2. The sequence (Lesson 1) begins 0, 2, 4, 6, 8, ...

odd numbers: Whole numbers that aren't even numbers. The sequence begins 1, 3, 5, 7, 9, ...

11A Rounding Whole Numbers and Decimals

It is easier to work with smaller numbers, which is why we like to use a process called *rounding*. A disadvantage of rounding is that we lose accuracy. For example, if you caught a bass that weighed 8.961 pounds on a certified scale, you might tell your friends you caught "about a 9 pound bass," since 8.961 is very close to 9. As long as you use the word "about," you aren't lying to say "9 pounds," which is a lot easier to discuss than "eight point nine six one."

Example 11.1 Round to the nearest tens place. a) 651 b) 87

solution: Follow the Lesson 11 Rule for rounding.

a) Circle the 5. Observe the digit to the right is 1, so we replace it with zero, like this: 651 >> **650**

b) Circle the 8. Observe the digit to the right is 7, so we replace it with a zero, and round the 8 up to 9, like this: 87 >> **90**

Example 11.2 Round to the nearest hundreds place. a) 350 b) 1,249

solution: Follow the Lesson 11 Rule for rounding.

a) Circle the 3. Observe the digit to the right is 5, so we replace it with 0, and round the 3 up to 4, like this: 350 >> **400**

b) Circle the 2. Observe the digit to the right is 4, so we replace it, and the 9, with 0's, like this: 1,249 >> **1,200**

Example 11.3 Round to the nearest thousands place. a) 9,900 b) 32,388

solution: Follow the Lesson 11 Rule for rounding.

a) Circle the 9 in the thousands place. Observe the digit to the right is 9, so we will "round up," but wait! We cannot round 9 any higher as a single digit. The rule says to add 1, so we do, and 9+1=10. Replace the digits to the right with 0's, like this: 9,900 >> **10,000**

b) Circle the 2, and observe the digit to the right is a 3, so we will "round down" to 32,000, like this: 32,388 >> **32,000**

Example 11.4 Round to the nearest whole number.   a) 22.9   b) 108.3

solution: Follow the Lesson 11 Rule for rounding. The decimal doesn't change anything! Just circle the ones place and look at the digit to the right, in the tenths place, to determine whether you round up or down.

a) Circle the 2 in the ones place. The digit to the right is 9, so round up to 23, like this: 22.9 >> **23**   Note that we didn't replace the 9 with a zero as in previous problems. That's because the problem asks to round to a whole number, so we write **23**, not 23.0.

b) Circle the 8 in the ones place. The digit to the right is 3, so we round down to 108, like this: 108.3 >> **108**

11B Divisibility

Rules for divisibility are listed above for 2 through 12. Arithmetic practice, including doing your daily facts, is probably the best way to get familiar with numbers and what they are or are not divisible by. Using the Rules would be a secondary tool, especially for larger numbers that you don't normally see in daily facts practice, math "flashcard" games, etc. Using a calculator is a third tool, although **we don't recommend calculator use for testing divisibility**. Use your own brain instead!

Some of the Rules for Divisibility are awkward and time-consuming to use, like 4, 7, 8 and 11. We are rarely going to use those in this course, although you are welcome to get familiar with them. The main Rules we will use include 2, 3, 5, 6, 9, 10 and 12. Two especially interesting rules include 3 and 9, where you sum the digits in a number and then divide by 3 or 9. It doesn't seem like that would work, but try it!

Example 11.5 Which number is divisible by 2?   32   209

solution: Study the definition for divisibility to understand what we mean by "divisible by." The Lesson 11 Rule is the last digit is even if the number is divisible by 2. Therefore, **32** is divisible by 2.

Example 11.6 Which number is divisible by 3? 57 82

solution: The Lesson 11 Rule is that the number is divisible by 3 if the sum of the digits is divisible by 3. Add the digits and find out!

57: 5+7 = 12, and 12÷3=4, so **57** is divisible by 3!
82: 8+2=10, and 10 is not divisible by 3, so 82 is not divisible by 3.

Example 11.7 Find the whole number factors for each of the following.
a) 14 b) 51

solution: Finding whole number factors means we are asking the question, like for a), “what times what equals 14?” Note that ALWAYS, the number times 1 is one pair of factors.

a) 14 and 1 are factors. 14 is an even number, so 2 is a factor, the other factor being 7 since 2×7=14. Therefore, the whole number factors are **1, 2, 7, and 14**.

b) 51 and 1 are factors. 51 is an odd number, so a good step is to add the digits, 5+1=6, and 6 is divisible by 3, so 3 is a factor. Perform long division to find the factor that pairs with 3:

$$\begin{array}{r} 17 \\ 3\overline{)51} \\ \underline{-3} \\ 21 \\ \underline{-21} \\ 0 \end{array}$$

Therefore, the whole number factors are **1, 3, 17, and 51**. Before you did this problem, did you know 3×17=51? Using the Rules for Divisibility helps make the unknown, known!

Practice Set 11

The subscripted number next to the problem number references which lesson the problem is from.

1_{11}. Round 1,565 to the nearest tens place.

2_{11}. Round 590 to the nearest hundreds place.

3_{11}. Round 2,249 to the nearest thousands place.

4_{11}. Round 164.8 to the nearest whole number.

5_{11}. Which number is divisible by 5?

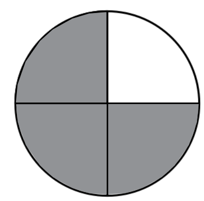
A) 45 B) 51 C) 504 D) 6

6_{11}. Find the whole number factors for 65.

7_{10}. Divide. Write the remainder as a whole number. 67÷5

8_{10}. Divide. Write the remainder as a fraction. 54/5

9_{10}. The following shape is divided into an equal number of parts. What fraction is NOT shaded?



10_{10}. What number is 1/5 of 67? Write answer as a mixed number.

11_{9}. Write three more fact families for 36÷9=4.

12_{9}. Divide 112 ÷ 4 Check using multiplication.

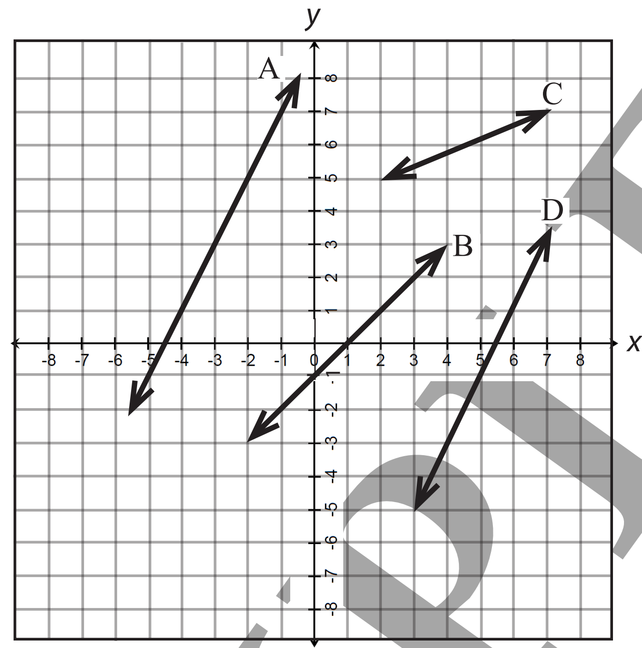
13_{9}. Divide. 600÷12 Check using multiplication.

14_{9}. Simplify. 3(8-2)-(8÷4)+10

15_{8}. Find the total cost for 2 dozen eggs that cost 30¢ each, and then convert to dollars.

16_{8}. Which of the following lines best fits the pattern created by the ordered pairs in the table?

x	y
-1	-2
0	-1
1	0
2	1



17_{7}. Find the missing number. $k + 9 = 21$

18_{7}. Find the missing number. $31 - x = 18$

19_{6}. If  $= x$, what does  equal?

20_{6}. Evaluate y-x if x=4 and y=11.

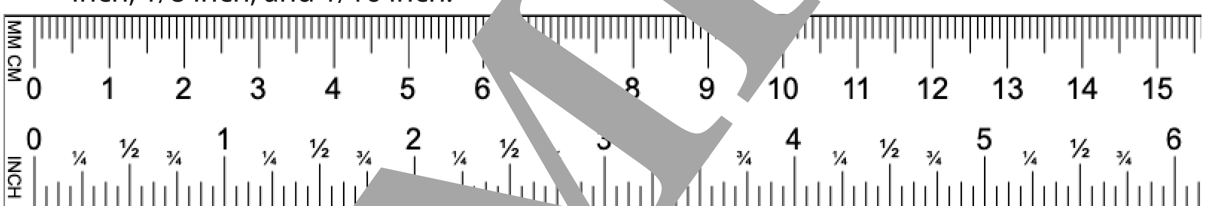
Lesson 12 Lines and Linear Measure; Decimal Number Lines

Review: Lessons 2, 8, 10, 11

Rules

For rounding a mixed number to the nearest whole number: If the fraction is 1/2 or greater, round up. Otherwise, leave the whole number as-is. Note: If it's just a fraction, then use the same rule and either round up to 1, or down to 0.

Ruler:
Below is an almost-accurate inch and metric ruler (Source: https://www.ginifab.com). On the metric side, large tick marks are 1 cm =10 mm apart, medium tick marks are 0.5 cm = 5mm apart, and small tick marks are 0.1 cm = 1 mm apart. On the English side, large tick marks represent 1 inch increments, followed by 1/2 inch, 1/4 and 3/4 inch, 1/8 inch, and 1/16 inch.



Decimal Number Rule: ANY number can be written as a decimal number by adding ".0" on the end. For example, 3 can be written 3.0, 100 can be written 100.0, etc. You can add as many 0's after the decimal point as you want, depending on the situation.

Definitions

plane: A flat surface having length and width only.

line segment: A line with a start point and end point.

ray: A line with a starting point but no end point.

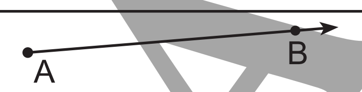
unit: In mathematics, *units* refer to fixed quantities used as standards of measurement. For example, a length might be measured using units of inches, centimeters, feet, etc.
decimal number: A number with a decimal point, followed by digits to show the fractional part. Examples include 40.5, 3.0, and 14.99.

12A Lines and Linear Measure

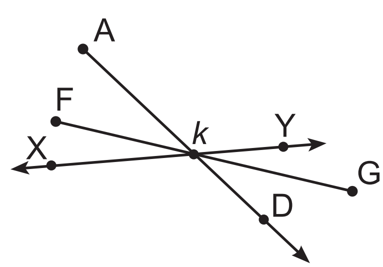
We defined *point* and *line* in Lesson 8. We used the same definitions that the famous math teacher, Euclid, wrote about over 2,300 years ago! Like numbers (Lesson 2), the shapes in Lesson 12 are *ideas*. We use a pen or pencil to draw real points and lines, and we draw them on a real plane. Our real plane is normally a piece of paper. For some shapes, like rays and lines, we can't draw the whole thing on paper, so we use an arrow tip on one (ray) or both (line) ends. The idea is that sides with an arrow go on forever!

We have to use our imagination, because compared to "goes on for ever," our paper is really small. The same goes for a point. The idea is that it "has no part," which means it is so small there is really nothing there! But we have to describe it somehow, so we draw a little dot on our paper. Sometimes our dot is bigger, sometimes smaller. That's okay! Just draw it, but also imagine that it is so small you can't really see its location, or for arrow tips, imagine they extend forever in that direction.

In Lesson 12, we will explore lines further, studying types of lines like line segments and rays. Study the following chart on how to draw and label line types using two letters. Note that order doesn't matter for lines or line segments, but for rays we always name the side with the point first.

Shape	Write it	Say it
• A	point A	point A
	$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$,	line AB, or line BA
	$\overrightarrow{AB}$	ray AB
	$\overline{AB}$ or $\overline{BA}$	segment AB, or segment BA

Example 12.1 Look at the lines shown. Write their names. When lines intersect, we use a point to describe the intersection. Write the point's name, too.



solution: Sometimes, we use the word "line" to describe more than one type of line. Here, we have line segment FG, line XY, and ray AD (not DA).

They intersect at point k. We will write them as follows:

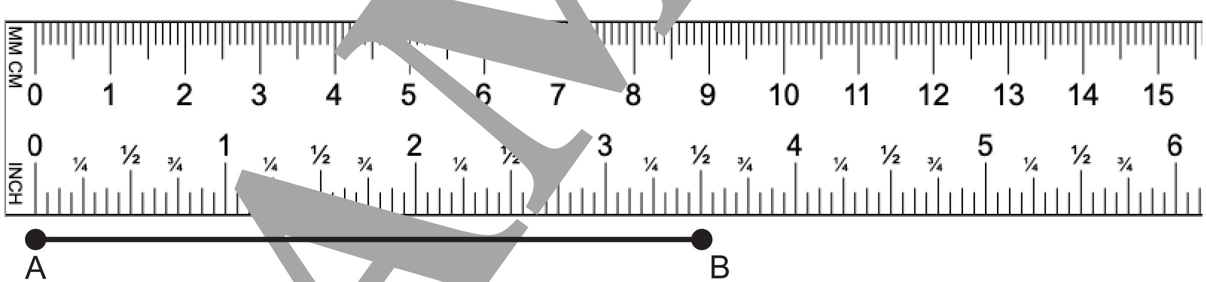
$\overleftrightarrow{XY}$, $\overline{FG}$, $\overrightarrow{AD}$, and **point *k***.

Next, we will use a ruler to measure some line segments. For problems from Lesson 12, all measurements will be in line with a 1/4, 1/2 or 3/4 inch mark. We will measure using smaller marks in Lesson 23. You will also be asked to round the measurement to the nearest whole number of centimeters (cm). These problems are designed to give you practice measuring, rounding, writing units and writing mixed numbers.

We are practicing measuring in centimeters (cm) and inches (in) because those are two of the most common types of length measurements you will encounter in everyday life.

IMPORTANT NOTE (write this down): For any measurement you do, **always** write the units.

Example 12.2 Measure segment AB using inches (in). Record the result as both a mixed number, and rounded to the nearest centimeter (cm).



solution: When we measure things with rulers, we normally line up one end at 0. That way, we don't have to do any subtraction, we just identify where the other end is located and record. In Ex. 12.2, notice A is at 0, which means we just need to identify where B is. If we start at A and move right, observe that we can count 1. . . 2. . . 3 inches, plus a little more. Do you see how B is located at the 1/2 inch mark? Therefore, segment AB equals $\boxed{3\frac{1}{2}\ in}$.

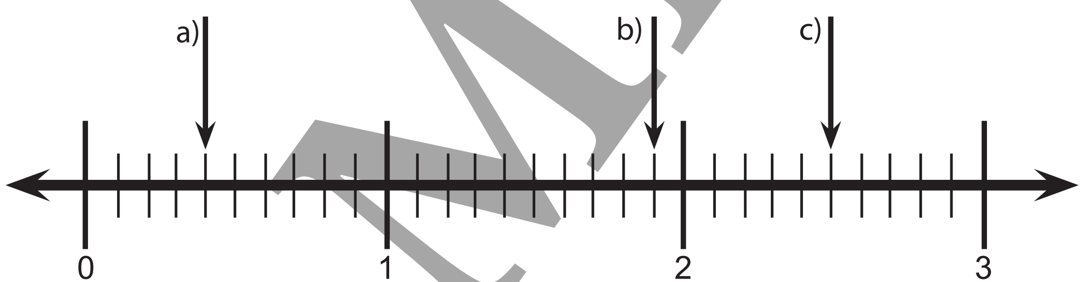
Next, observe the location of point B relative to the centimeter (cm) side. Do you see how segment AB is about **9 cm** long? Sometimes, we use a "wavy equals sign" to describe approximate values. We write $\overline{AB}$ **≈ 9 cm**. Remember, always write the units. If someone asked "how long is segment

AB?", and you said "nine," that doesn't tell them everything they need to know, because the segment is not nine inches long, it's **9 cm** long. Always include units when you measure something!

12B Decimal Number Lines

In Lesson 2, we used number lines to compare number types. The number lines we used normally had 0 in the middle, and "tick marks" spaced 1 apart. But, the tick marks do not have to be spaced 1 apart. In fact, rulers are a type of number line, and we just finished Ex. 12.2 where the inch ruler tick marks are spaced a fractional distance apart. And we didn't mention it, but the centimeter ruler had marks spaced 1 mm = 0.1 cm apart. If rulers can have decimal spacing, so can number lines!

Example 12.3 Which number is each arrow pointing to?

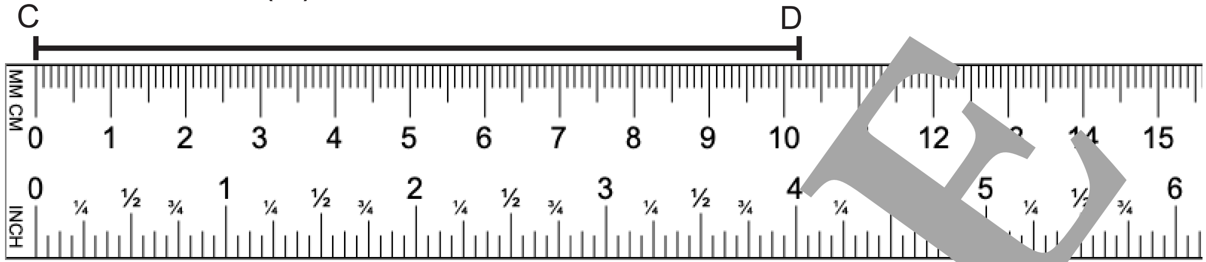


solution: Observe this is a decimal number line, with tick marks spaced 0.1 apart. Writing the marks as a sequence, we have 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, etc. Now that we know the pattern, we can identify each location:

a) = 0.4 b)=1.9 c) = 2.5

Note that for a), we wrote 0.4 and not .4. Both are okay, but in this course we will include the 0 in front. If we wanted to say each number, we would say them "four-tenths, one and nine-tenths, and two and five-tenths." Since 2.5 is halfway between two and three, we also say "two and a half." Finally, another way to say each number is "point four, one point nine, and two point 5."

Example 12.4 Measure segment CD using centimeters. Record the result as both a decimal number of centimeters (cm) , and rounded to the nearest inch (in).



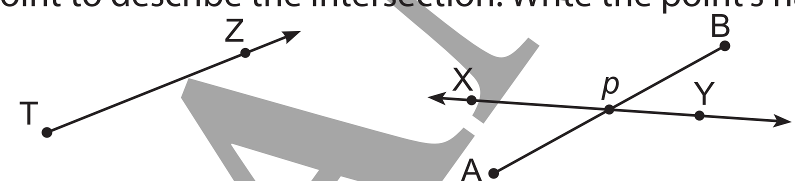
solution: As in Ex. 12.2, the left end (C) is lined up with 0. D is 2 tick marks to the right of 10 cm, so we can say segment **CD = 10.2 cm**.

We will use the same “wavy equals sign” as in Ex. 12.2 to show we are estimating. Viewing the ruler’s inch side, we see that $\overline{CD}$ **≈ 4 in**.

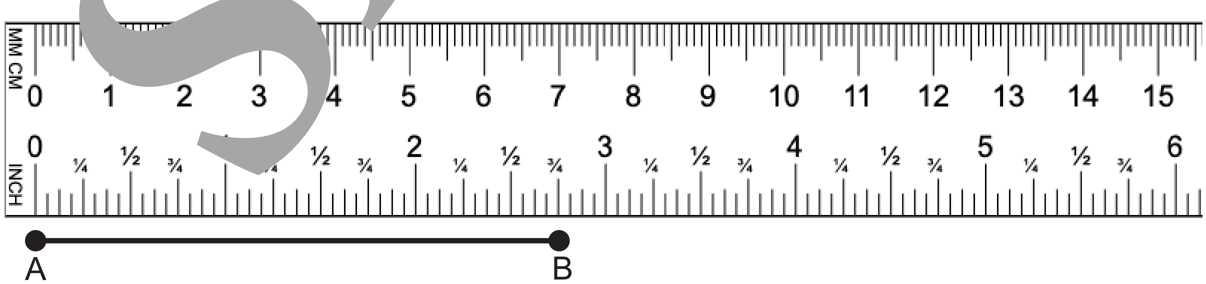
Practice Set 12

The subscripted number next to the problem number references which lesson the problem is from.

1_{12}. Look at the lines shown. Write their names. When lines intersect, we use a point to describe the intersection. Write the point’s name, too.



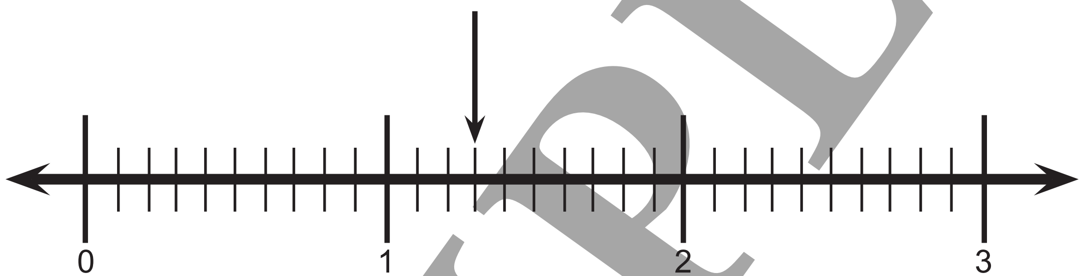
2_{12}. Measure segment AB using inches (in). Record the result as a mixed number.



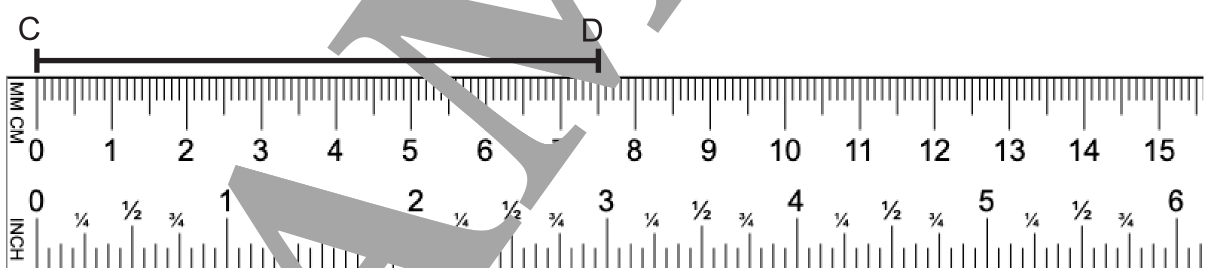
3_{12}. Measure segment AB again (see Problem 2), and this time record your measurement rounded to the nearest centimeter (cm).

4_{12}.Which number is the arrow pointing to?

5_{12}. Which number is the arrow pointing to?



6_{12}. Measure segment CD using centimeters. Record the result as a decimal number of centimeters (cm).



7_{12}. Measure segment CD again (see Problem 6), and this time record your measurement rounded to the nearest inch (in).

8_{11}. Round 1,949 to the nearest hundreds place.

9_{11}. Round 21,397 to the nearest thousands place.

10_{11}. Which number is divisible by 10?
A) 511 B) 51 C) 500 D) 6,005

11_{11}. Find the whole number factors for 27.

12_{10}. Divide. Write the remainder as a whole number. $21 \div 4$

13_{10}. Divide. Write the remainder as a fraction. 43/5

14_{9}. Write three more fact families for $4 \times 12 = 48$.

15_{9}. Divide and check. $4\overline{)56}$

16_{8}. Multiply.

$$\begin{array}{r} 24 \\ \underline{\times\ 27} \end{array}$$

17_{7}. Find the missing number. $9 + 11 = x$

18_{4}. Subtract.

$$\begin{array}{r} 5,111 \\ \underline{-\ 2,890} \end{array}$$

19_{4}. Add and subtract. Simplify inside parentheses first. 5-(7-4)+10-1

20_{2}. Compare 0 □ -4. Use a number line to help you.

Lesson 13 Missing Numbers in Multiplication and Division

Review: Lessons 6,7, 8, 9

Rules

For **multiplication with a missing number**, remember these rules:

1) If the product is missing, use multiplication to solve.
2) If either factor is missing, use division to solve.

For **division with a missing number**, remember two rules:

1) If the dividend is missing, use multiplication to solve.
2) If the divisor or quotient is missing, use division to solve.

No New Definitions

We covered missing numbers in addition and subtraction in Lessons 6 and 7, and will now add multiplication and division. We will do both equations (Lesson 7) and evaluations (Lesson 6), using fact families as needed to rearrange the equations.

Example 13.1 Find the missing number in each multiplication problem.

a) $8 \cdot 4 = x$ b) $6a = 30$ c) $c(4) = 24$

solution: Recall from Lesson 7 that standard form for solving equations is to finish with the variable on the left side of the equals sign.

a) This is the easiest type to solve since we know both factors. Simply multiply to see **x=32**. From the problem, it looks like we should write 32=x, but remember, standard form is to have the variable on the left, so we write **x=12**

For both b) and c), we apply the Lesson 13 rules and rearrange into a division fact, noting that the product of multiplication becomes the dividend in division:

b)	$6a = 30$	c)	$c(4) = 24$
	$a=30\div6$		$c=24\div4$
	$\boldsymbol{a=5}$		$\boldsymbol{c=6}$

Example 13.2 Find the missing number in each division problem.

a) $28 \div 7 = r$ b) $40 \div s = 5$ c) $\frac{t}{17} = 2$

solution: Apply the Lesson 13 Rules for division.

a) This is the easiest type to solve, so simply divide to see ***r*=4**.

b) The divisor is missing, so switch the divisor and quotient and solve:

$40 \div s = 5$
$40 \div 5 = s$
***s*=8**

c) Fractions are a form of division (see Lesson 9)! The dividend is missing, so rewrite the equation as a multiplication fact:

$t \div 17 = 2$
$t = 17 \times 2$
***t*=34**

Example 13.3 Evaluate each expression when a=1 and b = 2.

a) $a(b)$ b) $b \div a$

solution: Replace a with 1 and b with 2, and solve:

a) $a(b) = 1(2) =$ **2**

b) $b \div a = 2 \div 1 =$ **2**

Example 13.4 Evaluate each expression when x = 1 and y = 8.

a) $3xy$ b) $\frac{y}{2x}$

solution:

a) Recall from Ex. 6.4 that we have already done some evaluations with multiplication. In Ex. 6.4, we evaluated 2y, noting that 2y="two times y." In the same way, 3xy="three times x times y." Substitute in and multiply in pairs:

$3xy=3(1)(8)=$
$3(8) =$ **24**

b) We could rewrite this as long division, but Lesson 13 problems will normally have fairly simple division, using numbers you should already be familiar with. Instead of leaving as a fraction, we will rewrite it using the "÷" symbol:
$y \div 2x =$
$8 \div 2(1) =$
$8 \div 2 =$ **4**

Practice Set 13

The subscripted number next to the problem number references which lesson the problem is from.

1_{13}. Find the missing number. $7f = 35$

2_{13}. Find the missing number. $g \cdot 3 = 27$

3_{13}. Find the missing number. $90 \div a = 9$

4_{13}. Find the missing number. $\frac{b}{8} = 6$

5_{13}. Evaluate $b \times a$ when $a=6$ and $b = 3$.

6_{13}. Evaluate a/b when $a=6$ and $b = 3$.

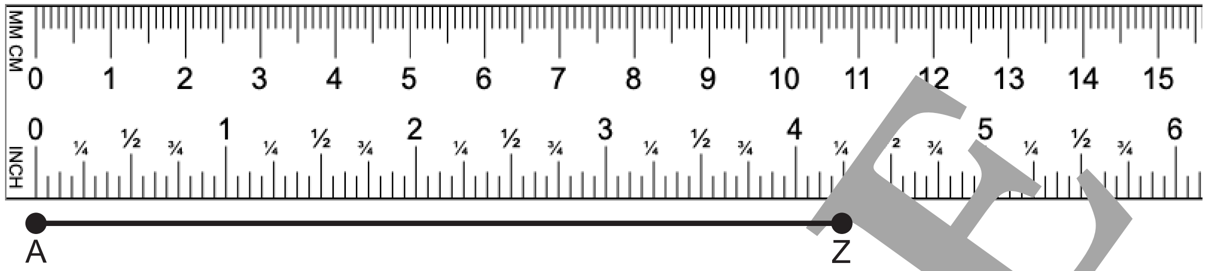
7_{13}. Evaluate $4xy$ when $x=1$ and $y=7$.

8_{13}. Evaluate $\frac{14x}{y}$ when $x=1$ and $y=7$.

9_{12}. Which of the following best describes ray EF?

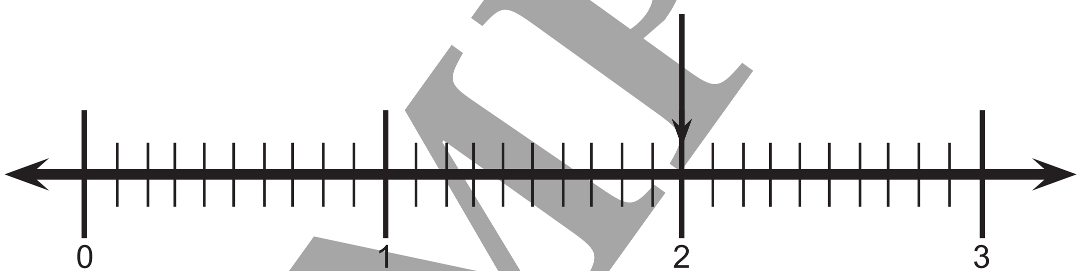
A) $\overline{EF}$ B) EF C) $\overleftrightarrow{EF}$ D) $\overrightarrow{EF}$

10_{12}. Measure segment AZ using inches (in). Record the result as a mixed number.



11_{12}. Estimate the length of segment AZ in Problem 10 using centimeters (cm). Record the result as a whole number of centimeters.

12_{12}. Which number is the arrow pointing to? Write answer as a decimal number. Hint: Apply the Decimal Number Rule in the Lesson 12 Rules.



13_{11}. Round 49 to the nearest tens place.

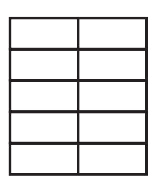
14_{11}. Round 49.4 the nearest whole number.

15_{11}. Which number is divisible by 6?

A) 46 B) 56 C) 66 D) 76

16_{11}. Find the whole number factors for 7.

17_{10}. Shade 2/5 of the figure shown.



18_{9}. Divide and check. $600 \div 24$

19_{8}. Find the product. $\begin{array}{r} 465 \\ \underline{\times 201} \end{array}$

20_{7}. Find the missing number. $5+x=11$

Lesson 14 Story Problems About Addition and Subtraction

Review: Lessons 3, 4, 7

Rules

For solving ANY story problem, start by asking two questions:

1) What am I solving for?

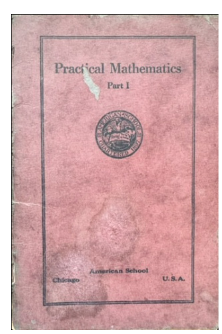
2) What do I know?

For Step 3, create an equation and solve. Look for key words that help you determine what kind of equation to create. Check your work when possible.

No New Definitions

Recall from Lesson 1 that we defined math as "the language of science." But what is *science*? When people "do science," what they are really doing is making observations about God's creation. They are asking questions, performing experiments, and drawing conclusions about things God made, all in an effort to know more. Math is also about measuring things, about finding out how much or how little we have of a certain quantity. But again, we are measuring things God created, or perhaps things that man made from things that God created!

What we are talking about is practical mathematics, or what some people call "real world" math. This is more than just knowing your arithmetic facts or the steps of long division. Practical mathematics is about using those skills to help you solve problems in your everyday life.



Many times, practical mathematics problems are are presented in the form of a story, or word problem. Here, you will apply your addition and subtraction skills to solve story problems. Some of these problems will be from a book titled *Practical Mathematics*, published over 100 years ago in the United States! The letters "PM" in parentheses will let you know if you are solving a problem from *Practical Mathematics*. It is good for us to learn from history, to see what students were expected to know who would go on to apply math in their everyday lives, just like you will!

Example 14.1 Peter retrieved 116 fish from the net, and John retrieved 37. How many fish did they retrieve altogether? Create an equation and solve.

solution: Follow the 3 steps in the Rules:

1) What are you solving for? You are solving for the total number of fish retrieved.

2) What do you know? You know Peter caught 116 fish, and John caught 37 fish.

3) A key word is *altogether*, which directs us to create an addition equation. Think about this though; right now, the total number of fish is missing from our story, so let's create an equation with a missing number, N, like the equations in Lesson 7:

N = 116 + 37 Rewrite vertically and solve:

$$\begin{array}{r} \overset{1}{1}16 \\ +\ \ 37 \\ \hline \mathbf{153} \end{array}$$

Did you know that in John 21, Peter and some of the disciples went fishing, catching 153 fish altogether? The Bible doesn't say who retrieved more fish from the net, but it is very specific about the total number of fish!

Example 14.2 (PM) A marine engine during a 3 hours' run makes 9,187 revolutions the first hour, 9,062 the second, and 9,233 the third. How many revolutions does it make in the 3 hours?

solution: Follow the 3 steps in the Rules:

1) What are you solving for? You are solving for the total number revolutions made in 3 hours.

2) What do you know? You know how many revolutions the engine made in each of 3 hours.

3) A key phrase is *how many*, which directs us to create an addition equation. We have three numbers to add, which you've done several times since Lesson 3:

$$\begin{array}{r} 9{,}187 \\ 9{,}062 \\ +\ 9{,}233 \\ \hline \mathbf{27{,}482} \end{array}$$

One challenge when we add more than two numbers is that we cannot check our work using subtraction. However, we can estimate! For example, observe each number is about 9,000, and 3×9,000=27,000, so our actual answer should be close to this, and it is.

Example 14.3 Robert purchased a total of 50 apples. When he got home, he realized 15 apples were not ripe. How many apples were ripe?

solution: Follow the 3 steps in the Rules, noting this is a subtraction problem:

1) What are you solving for? You are solving for the number of ripe apples.
2) What do you know? You know Robert purchased a total of 50 apples, and 15 were not ripe.
3) Some "were not" ripe, which is a hint the number of ripe apples is less than 50, or 50 - 15 = N. We can rewrite vertically and solve:

$$\begin{array}{r} \overset{4}{5}0 \\ -\ 15 \\ \hline \mathbf{35} \end{array}$$

Check with addition:

$$\begin{array}{r} 35 \\ +\ 15 \\ \hline 50 \end{array}$$

Therefore, there were **35 ripe apples**.

Example 14.4 The basket was full of flowers when Emily was finished picking. After she gave 21 flowers to a friend, she had 54 left. How many flowers did Emily pick altogether?

solution: Follow the steps in the Rules. You may need to consider Lesson 9 when creating an equation:

1) What are you solving for? You are solving for the number of flowers when the basket was full.
2) What do you know? You know Emily gave 21 to a friend, which left her with 54. 54 is the difference therefore, between the total and 21.
3) The "full" amount is missing, which means the largest number, or minuend, is missing. "54 left" is also a key phrase, because it tells us something was subtracted and some were left. From Lesson 9, we know that when the minuend is missing, we can use addition to

solve. Let's write this one out carefully:

minuend - subtrahend = difference
N - 21 = 54   rearrange as an addition problem:

$$\begin{array}{r} 21 \\ +\ 54 \\ \hline \mathbf{75} \end{array}$$

Check with subtraction:

$$\begin{array}{r} 75 \\ -\ 54 \\ \hline 21 \end{array}$$

Therefore, Emily picked **75 flowers** altogether.

Example 14.5 (PM) From a tank containing 935 gallons of water, 648 gallons were drawn off. Then 247 gallons ran in. How many gallons were then in the tank? (Hint: Subtract 648 from 935 and add 247).

solution: We follow the steps in the Rules, but this problem comes with a hint, so we should simply apply it! First, we will subtract, and then add back in 247 gallons. We could write the problem as a single expression by using parentheses, as we did in problems like Ex. 4.4:

(935-648)+247   We simplify inside parentheses first, then add 247:

$$\begin{array}{r} {}^{8}\ {}^{12}\ \\ 935 \\ -\ 648 \\ \hline 287 \end{array}$$

Now, add 287+247:

$$\begin{array}{r} 287 \\ +\ 247 \\ \hline \mathbf{534} \end{array}$$

Therefore the tank ended with **534 gallons** stored.

Practice Set 14

The subscripted number next to the problem number references which lesson the problem is from.

1_{14}. 2 Samuel 24:9 says "in Israel there were 800,000 valiant men who drew the sword, and the men of Judah were 500,000." How many men is this altogether? Create an equation and solve.

2_{14}. (PM) Coal is fed to a furnace as follows: Monday, 376 pounds; Tuesday, 307 pounds, Wednesday 438 pounds. Find the total for Monday-Wednesday.

3_{14}. Adrian purchased a total of 70 blackberries. He ate 23 of them on the way home. How many blackberries did he have left?

4_{14}. When the store opened, the cash register was full of $20 bills. 86 were removed for sales, leaving 14 at closing time. How many $20 bills were in the register when the store opened?

5_{14}. (PM) A man purchased 8,983 bricks, but used only 5,363. How many did he have left?

6_{13}. Find the missing number. $2 \cdot 9 = x$

7_{13}. Find the missing number. $5a = 60$

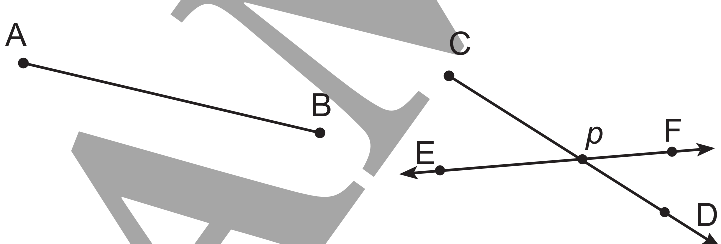
8_{13}. Find the missing number. $42 \div 6 = s$

9_{13}. Find the missing number. $54 \div t = 6$

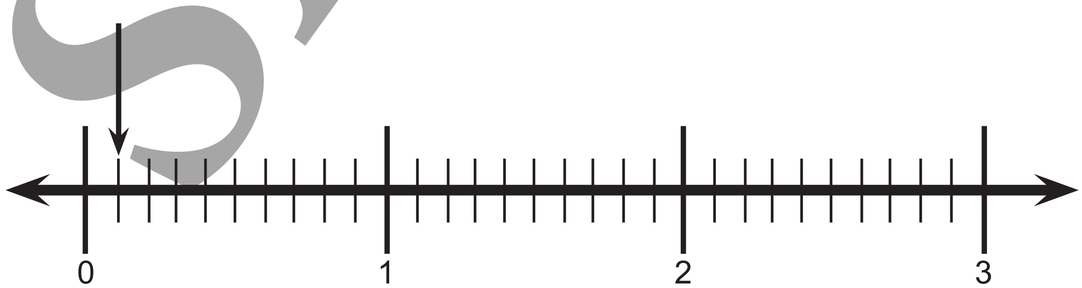
10_{13}. Evaluate $a(b)$ when $a=4$ and $b = 5$

11_{13}. Evaluate b/a when $a=3$ and $b = 12$

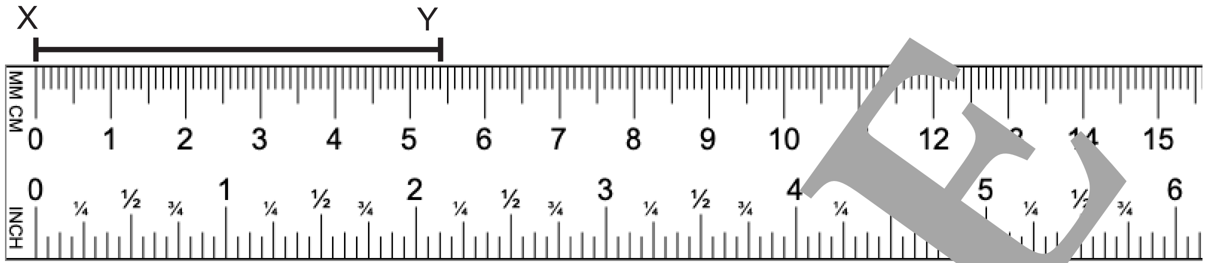
12_{12}. Look at the types of lines shown. Write their names. When lines intersect, we use a point to describe the intersection. Write the point's name, too.



13_{12}. Which number is the arrow pointing to?



14_{12}. Measure segment XY using centimeters. Record the result as a decimal number of centimeters (cm).



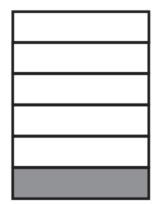
15_{12}. Use Problem 14 to estimate the length of segment XY, rounded to the nearest inch.

16_{11}. Round 557 the nearest tens place.

17_{11}. Which number is divisible by 9? 29 81

18_{11}. Find the whole number factors of 25.

19_{10}. What fraction of the following shape is shaded?



20_{2}. Find the numerical value of the 9 in 693,288,157.

Lesson 15 Story Problems About Combining and Separating

Review: Lessons 3, 4, 14

No New Rules or Definitions

Addition and subtraction word, or story problems, like those in Lesson 14, can have a variety of key words and phrases for describing addition and subtraction. Lesson 15 simply expands on Lesson 14 by adding more types of addition (combining) and subtraction (separating) word problems. Sometimes, you hear these problem types referred to as "some and some more"(combining), and "some and some went away" (separating) problems.

Example 15.1 Cecilia counted diatoms in a water sample. In the first microscope slide, she counted 147 diatoms. After the second slide, the counter showed a total of 305 diatoms. How many diatoms did Cecilia count on the second slide?

solution: Refer to the Lesson 14 Rules for the steps to solving story problems. Here, we want to find the number of diatoms Cecilia counted on the second slide. We know the first slide, the "some" part (147), and the total (305), we just don't know the "some more" part. We could describe everything using variables, like this:

some + some more = total, or more descriptively as
slide 1 + slide 2 = total

S1 + S2 = T

Rearrange into a subtraction fact to solve for S2:

$$S2 = T - S1 \qquad \begin{array}{r} \overset{2}{3}\overset{9}{\cancel{0}}\overset{1}{5} \\ -\ \ 147 \\ \hline \mathbf{158} \end{array} \qquad \text{check: } \begin{array}{r} \overset{1}{1}\overset{1}{5}8 \\ +\ \ 147 \\ \hline 305 \end{array}$$

Therefore, slide 2 had **158 diatoms**.

Example 15.2 Emily had a basket filled with 75 flowers. She gave some to her friend. 54 flowers remained. How many flowers did Emily give to her friend?

solution: We want to know how many flowers Emily gave her friend. We know the basket had 75 flowers, and "some went away", leaving 54 flowers. The phrase *gave some* tells us to set up a subtraction problem. We can use variables to write:

some - some went away = left over, or
total - friend's = remaining, or
T - F = R, which we rearrange as another subtraction fact, F = T - R, and solve:

$$\begin{array}{r} 75 \\ -\ \underline{\ \ 54} \\ \mathbf{21} \end{array} \qquad \text{Check:} \quad \begin{array}{r} 21 \\ +\ \underline{\ \ 54} \\ 75 \end{array}$$

Therefore, Emily gave her friend **21 flowers**. Did you notice this is the same number facts as Ex. 14.4? This time, we just solved for the number of flowers Emily gave her friend, which was given in Ex. 14.4.

Example 15.3 Arthur had $50.00. He went to the sporting goods store and returned home with $33.76. How much did Arthur spend at the store?

solution: This is a similar problem to Ex. 15.2, except with money. We know a total amount (T) and a remaining amount (R), we just need to find the "separated" amount, which is the amount spent at the sporting goods store (S). We create a similar equation to Ex. 15.2 and solve for S:

some - some went away = left over, or
total - spent = remaining, or
T - S = R, which we rearrange as another subtraction fact, S = T - R, and solve:

$$\begin{array}{r} {}^{4\,9\,9} \\ 50.00 \\ -\ \underline{\ \ 33.76} \\ \mathbf{16.24} \end{array} \qquad \text{Check:} \quad \begin{array}{r} 33.76 \\ +\ \underline{\ \ 16.24} \\ 50.00 \end{array}$$

Therefore, Arthur spent **$16.24** at the sporting goods store.

Practice Set 15

The subscripted number next to the problem number references which lesson the problem is from.

1_{15}. The Texas Aggies scored 17 points in the first half of their football game. They finished the game with 41 points. How many points did the Aggies score in the second half?

2_{15}. The reservoir contained 897.5 acre-feet of water. Some was diverted for irrigation. How much was diverted if 606.7 acre-feet were diverted?

3_{15}. Naomi had \$100.00. He went to the retail store and returned home with \$47.55. How much did Naomi spend at the store?

4_{14}. Ranger shot 110 rounds and Scout shot 267 rounds. How many rounds did they shoot altogether?

5_{14}. 150 sockeye salmon were resting below a waterfall. 63 of them had changed color from chrome bright to red. How many sockeye were still chrome bright?

6_{14}. The basket was full of eggs when Hadassah was finished collecting. After she gave 13 eggs to a neighbor, she had 35 left. How many eggs did Hadassah collect altogether?

7_{13}. Find the missing number. $d(6) = 36$

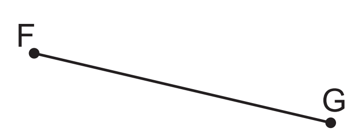
8_{13}. Find the missing number. $\frac{m}{19} = 1$

9_{13}. Evaluate $5yz$ when $y = 2$ and $z = 8$.

10_{13}. Evaluate 3z/y when $y = 2$ and $z = 8$.

11_{12}. Which of the following best describes the type of line shown?

A) $\overline{FG}$ B) $\overleftarrow{FG}$ C) $\overleftrightarrow{FG}$ D) $\overrightarrow{FG}$



12_{11}. Round 2,749 to the nearest hundreds place.

13_{11}. Which number is divisible by 3? 41 42

14_{10}. Divide. Write the remainder as a whole number. 12÷8

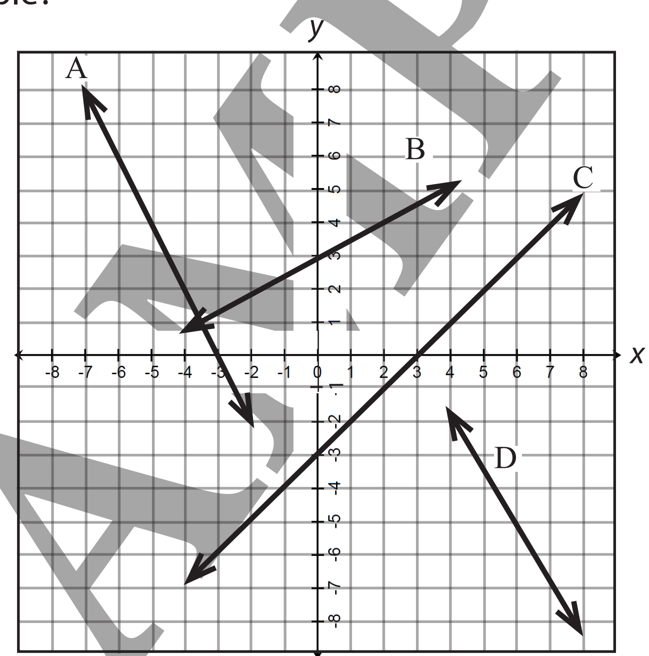
15_{10}. What number is 1/4 of 125? Write answer as a mixed number.

16_{9}. Simplify. (4÷4)2+10-3

17_{8}. Find the total cost for 3 dozen papayas that cost 83¢ each, and then convert to dollars.

18_{8}. Which of the following lines best fits the pattern created by the ordered pairs in the table?

x	y
-2	-5
0	-3
2	-1
4	1



19_{6}. Evaluate 3y-2x when x = 3 and y = 2.

20_{5}. Use the map in Lesson 5A to estimate the location of Anchorage, AK.